Statistiek

Deel 1: Herhaling

(1) $\bar{x} = 9825,2$ $\qquad n = 10$

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

$92\,1\cancel{\ }600 + 715\,152 + 104\,760 + 866\,140 + 2254\,001$
$+ 60\,844 + 3806\,401 + 18\,860 + 308\,765 +$
$+ 1\,331\,716$
$= \cancel{1\,111}\ 11\ 119\ 955$ ~

OF
$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}$$

✓

$= 1058839126 - 965345550,4 = 10\ 388\ 241,73$

(2) a) $6,6$
b) $s^2 = \frac{235 - 217,8}{4} = 4,3$

$\rightarrow s = 2,07$

(3) ~ a) $P[0 < x < 0,8] = 0,2881$
✓ b) $0,3849 + 0,5$
✓ c) $0,1554$
✓ d) $0,1915 \cdot 2 = 0,383$
✓ e) $0,5 - 0,4452 = 0,0548$
✓ f) $0,5 - 0,3413 = 0,1587$
✓ g) $0,5 + 0,1179 = 0,6179$
✓ h) $0,1327$
✓ i) $0,3313$
✓ j) $0,2857$

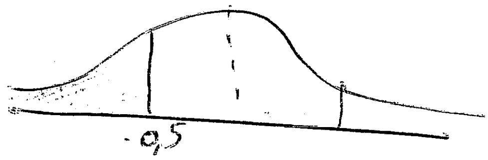
(4) $\bar{x} = 30$

$s = 10$

a) $P[30 < y < 36] =$ ~~30-30~~

$= P\left[0 < z < \frac{36-30}{10}\right] = 0{,}2257$

b) $\left[y < 25\right] = \left[z < \frac{25-30}{10}\right] = z < -0{,}5$



$= 0{,}5 - 0{,}\cancel{2257}\,1915 = 0{,}3085$

c) $\left[y > 50\right] = P\left[z > \frac{50-30}{10}\right] = P[z > 2]$

$= 0{,}0228$

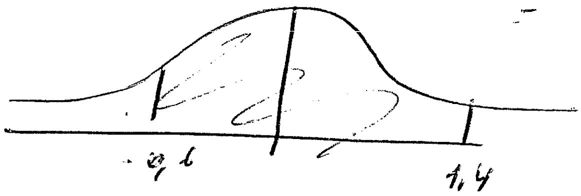
d) $P[35 < y < 45]$

$= P\left[\frac{35-30}{10} < z < \frac{45-30}{10}\right] = 0{,}5 < z < 1{,}5$

$= 0{,}2417$

e) $P[24 < y < 44] = P\left[\frac{24-30}{10} < z < \frac{44-30}{10}\right]$

$= [-0{,}6 < z < 1{,}4]$



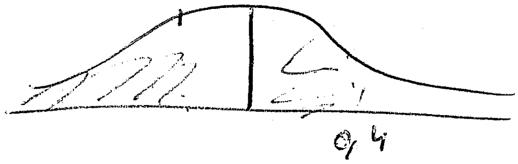
$= 0{,}6449$

5

a) $P[0 < x < A] = 0{,}45$

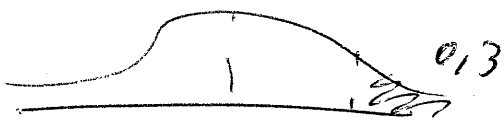
$A = 1{,}645$

b) $P[x < B] = 0{,}9$



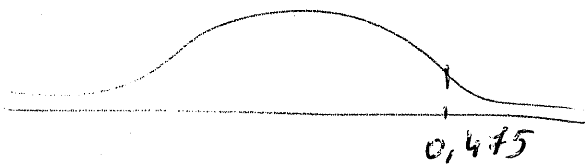
$= 1{,}28$

c) $P[x > C] = 0{,}3$



$= 0{,}52$

d) $P[-D < x < D] = 0{,}95$



$= 1{,}96$

e) $P[-E < x < E] = 0{,}99$

$0{,}495$

$= 0{,}2575$

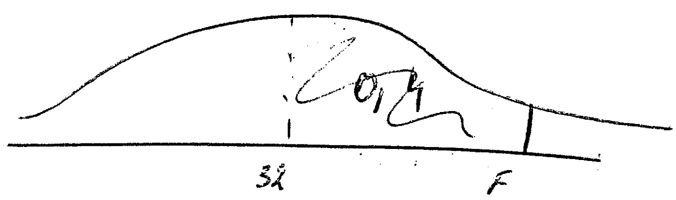
6 $\mu = 32$

$\sigma = 8$

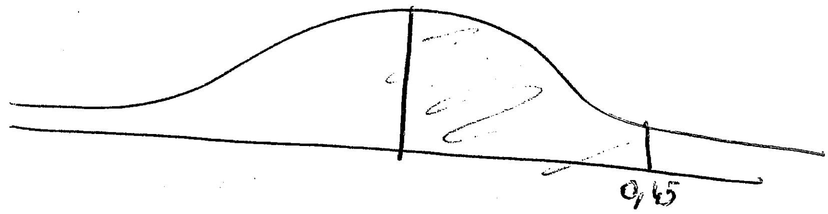
$P[0 < Z < F] = 0{,}4 =$

$\frac{F - 32}{8} = \cancel{0{,}4}\ 1{,}28$

$F = 42{,}24$

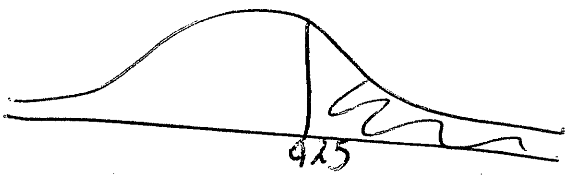


b) $P[S < G] = 0{,}95$



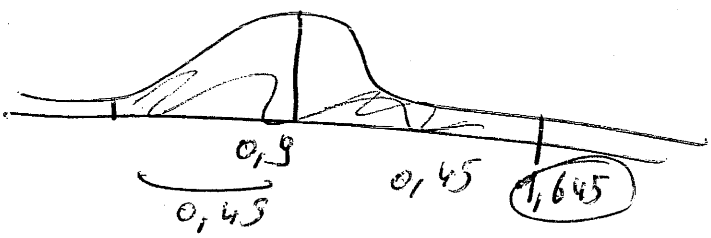
$\frac{G - 32}{8}$ 1,645 $= 45{,}16$

c) ~~K~~ $P[S > H] = 0{,}4$



P[~~≤S~~ $\frac{H - 32}{8} = 0{,}25$ $= 34$

d) $P[32 - I < S < 32 + I] = 0{,}9$



$\frac{\cancel{32} + I - \cancel{32}}{8} = 1{,}645$ $\Rightarrow 13{,}16$

e) $P[32 - J < S < 32 + J] = 0{,}99$ $\Rightarrow 2{,}575$

$= 0{,}495$

~~32+~~ $J = 2{,}575 \times 8 = 20{,}6$

Deel 2

11

B) 1) 95% σ bekend! → Z-score

a) $n = 16$ $\cancel{\text{t-score} = \frac{36-40}{4/\sqrt{16}} = 2,12}$

$\cancel{\bar{x} \pm t^{\alpha/2}_{n-1} \frac{s}{\sqrt{n}} = \bar{x} \pm t^{0,025}_{15} \frac{4}{4}}$

$\cancel{\rightarrow \bar{x} \pm 2,13 \cdot \frac{4}{4} = 33,88 < \mu < 38,12}$

$Z \pm \quad \bar{x} \pm 1,96 = \boxed{34,04 < \mu < 37,96}$

b) $n = 100 = \bar{x} \pm 1,96 \cdot \frac{4}{\sqrt{100}} = \boxed{35,216 < \mu < 36,784}$

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2) $n = 400$

$\bar{x} = 25000, \ s = 1600$

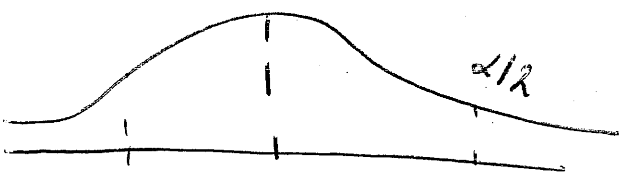
99,74% → 0,9974

→ $\alpha =$ 0,0026

→ $\alpha/2 = 0,0013$

$\rightarrow Z =$ 3,01

0,5 − 0,0013 = 0,4987



$\rightarrow 25000 \pm 3 \cdot \frac{1600}{\sqrt{400}} = [24760, 25240]$

③ $n = 200$

a) $\hat{p} = \frac{180}{200} = 0{,}90$

95%: $\left[0{,}90 \pm 1{,}96 \sqrt{\frac{0{,}9 \cdot 0{,}1}{200}}\right]$

$\Rightarrow \left[0{,}90 \pm 1{,}96 \sqrt{\frac{0{,}9 \cdot 0{,}1}{200}}\right]$

$= \left[0{,}90 \pm 0{,}0416\right]$

$\Rightarrow \left[0{,}8584 < p < 0{,}9416\right]$

b) $0{,}90 \pm 0{,}01$

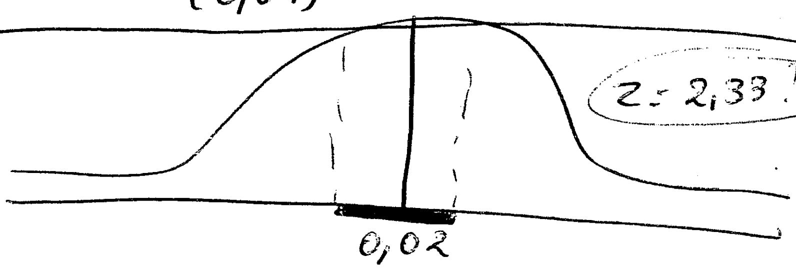
$\rightarrow \cancel{0{,}90}\; 0{,}01 = 1{,}96 \sqrt{\frac{0{,}9 \cdot 0{,}1}{200}}$

$0{,}005$

$Z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}} = B \Leftrightarrow n = \frac{(Z_{\alpha/2})^2 \cdot p(1-p)}{B^2}$

$= \frac{\cancel{(1{,}96)}^2 \cdot 0{,}09}{(0{,}01)^2}$

$$\boxed{\Rightarrow \frac{(1{,}96)^2 \cdot 0{,}09}{(0{,}01)^2} = 3458}$$



$z = 2{,}33$!

1) $n = 8$

$-22 \quad -25 \quad -28 \quad -20 \quad -38 \quad -35 \quad -32 \quad -40$

$\bar{x} = -30$

$$-30 \pm t_{7}^{0{,}025} \cdot \frac{s}{\sqrt{n}}$$

$$s^2 = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1} = \quad - \frac{\cancel{57600}}{8}$$

$$= \frac{1586 - 1200}{7} = 55{,}14$$

$\Rightarrow s = 7{,}43$

$$-30 \pm 2{,}365 \; \frac{7{,}43}{\sqrt{8}}$$

$= -30 \pm 2{,}365 \cdot 2{,}81$

$\Rightarrow -23{,}35 \longrightarrow -36{,}65$

$\rightsquigarrow$ t-score ?

Hoofdstuk 8

1) a) $H_0: \mu_0 \leq 150$

$H_a: \mu_0 > 150$ $\alpha = 0,01$

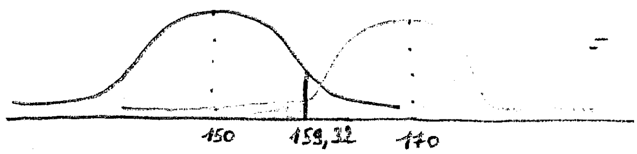
$$AG: \left]-\infty ; \mu_0 + z_\alpha \cdot \frac{s}{\sqrt{n}}\right] = \left]-\infty ; 150 + 2,33 \cdot \frac{20}{\sqrt{25}}\right]$$

$$= \left]-\infty ; 159,32\right]$$

$\bar{x} = 170 \notin AG \Rightarrow$ Op basis v/deze steekproef H_0 verwerpen.

b) Kans op fout 2e soort (Ho accepteren terwijl Ha juist is)

$$\beta = P\left[\bar{x} \in AG_{H_0} \mid \bar{x} : N\left(170 ; \frac{20}{\sqrt{25}}\right)\right] \Rightarrow P[\bar{x} < 159,32] = P\left[z < \frac{159,32 - 170}{4}\right]$$



$$= P[z < -2,67]$$

$$\Rightarrow \beta = 0,0038$$

2) $H_0: p \leq 0,10$

$H_a: p > 0,10$ $\alpha = 0,01$

$$AG = \left]-\infty ; p_0 + z_\alpha \cdot \sqrt{\frac{p_0 \cdot (1-p_0)}{n}}\right] = \left]-\infty ; 0,10 + 2,33 \cdot \sqrt{\frac{0,1 \cdot 0,9}{200}}\right] = \left]-\infty ; 0,15\right]$$

$\hat{p} = \frac{40}{200} = 0,2 \notin AG \Rightarrow H_0$ verwerpen

3) $H_0: \mu \geq 160$ $\bar{x} = 150$ $\alpha = 0,05$

$H_a: \mu < 160$ $s = 5$

$$P[\mu < 150]$$

$$\overset{STD}{\Leftrightarrow} P\left[z < \frac{150 - 160}{\frac{5}{\sqrt{6}}}\right] = P[z < -4,89] = \ldots$$

P-waarde zeer klein: < 0,001 → H_0 verwerpen

(4)

a) ~~$H_0: \mu = 15$~~ $\alpha = 0{,}05$

~~$H_A: \mu \neq 15$~~ $\alpha/2 = 0{,}025 \rightarrow \cancel{1{,}96}$ $\quad \dfrac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}{n-1}$

$t^{\alpha/2}_{n-1} \quad t^{0{,}025}_{7}$

$s^2 = \dfrac{1878 - 1800}{7} = 11{,}14$

BBI $= \left[\mu \pm t \cdot \dfrac{s}{\sqrt{n}}\right] = \left[15 \pm 2{,}365 \cdot \dfrac{3{,}34}{\sqrt{8}}\right]$

$= [12{,}21 \; ; \; 17{,}79]$

b) $H_0: \mu \geq 21$

$H_a: \mu < 21$ $\quad \rightarrow AG: \left[21 - t^{0{,}5}_{7} \cdot \dfrac{3{,}34}{\sqrt{8}} \; ; \; +\infty\right[$

$1{,}895$

$= [18{,}76 \; ; \; +\infty[$

~~Variantie bekend?~~ ~~verwerpen~~

~~H_0~~ $\bar{X} = 15 \rightarrow H_0$ verwerpen

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(1) $H_0: p \geq 0{,}98$

$H_A: p < 0{,}98$ $\quad \alpha = 0{,}01$

$AG: \left[p_0 - z_\alpha \cdot \sqrt{\dfrac{0{,}98 \cdot 0{,}02}{150}} \; ; \; +\infty\right[\; = \; [0{,}98 - 2{,}33 \cdot 0{,}0114 \; ; \; +\infty[$

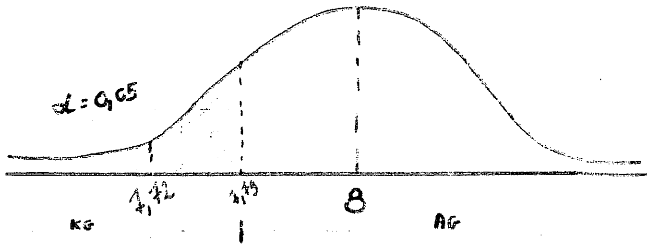
$= [0{,}953 \; ; \; +\infty[$

$\hat{p} = \dfrac{137}{150} = 0{,}913 \notin AG \rightarrow H_0$ verwerpen

Hoofdstuk 8: oef les 1

(6) a) $H_0: \mu \geq 8$ (want bevat gelijkheid) $\qquad \bar{x} = 7,72$

$H_a: \mu < 8 \qquad \sigma = 0,5$



$$AG = \left[\mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}}, +\infty\right[$$

$$= \left[8 - 1,645 \cdot \frac{0,5}{4}, +\infty\right[= [7,79; +\infty[$$

$\bar{x} = 7,72 \notin AG \Rightarrow H_0$ verwerpen en H_a aannemen

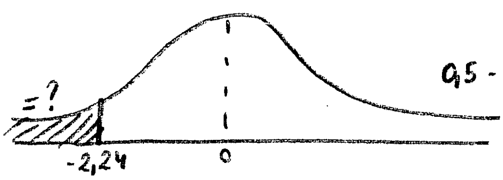
→ Producent is juist

σ geg? → z
n geg — $n \geq 30$ → z (cls) ; $n < 30$ → t

b) $P[\bar{x} < 7,72] = P\left[z < \frac{7,72 - 8}{\frac{0,5}{\sqrt{16}}}\right] = P[z < -2,24] = \boxed{0,0125 < \alpha}$

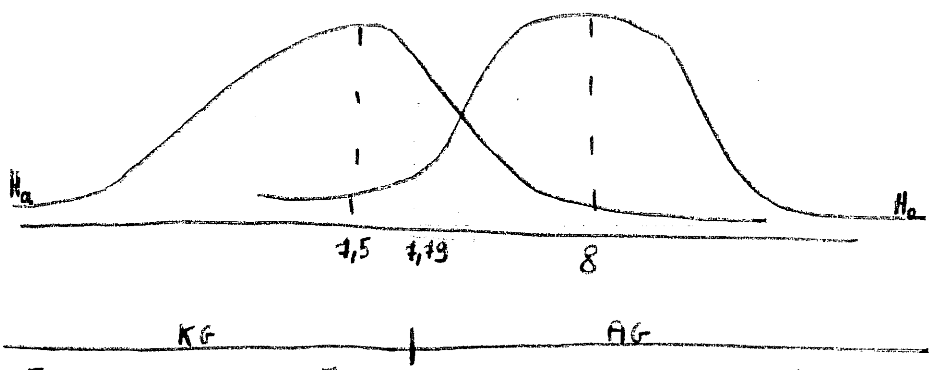
⇓

Producent is juist



0,5 - 0,4875 = 0,0125

c) Onderscheidingsvermogen: $1 - \beta$



: Fout 1e soort

: Fout 2e soort

$\rightarrow \beta = P[X \in AG_{H_0} \mid \mu_a = 7,5] = P[\bar{x} > 7,79 \mid \mu_a = 7,5] = P\left[z > \frac{7,79 - 7,5}{0,5/4}\right]$

$P[z > 2,32] = 0,0102 \Rightarrow$ Bij geldigheid H_A 98,98% kans om de juiste beslissing te nemen

⑤ a) → FRACTIES → P (pop. fractie)

$\alpha = 0,05$

$H_0: P \leq 0,1$

$H_a: P > 0,1$

$$AG = \left]-\infty ; P_0 + z_\alpha \sqrt{\frac{P_0 \cdot (1-P_0)}{n}}\right]$$

$$= \left]-\infty ; 0,1 + 1,645 \sqrt{\frac{0,1 \cdot 0,9}{60}}\right] = \left]-\infty ; 0,163\right]$$

$\hat{P} = 8/60 = 0,133 \in AG \rightarrow H_0$ ✓

b) $\beta = P\left[\hat{P} \in AG_{H_0} \mid P_A = 0,2\right] = P\left[\hat{P} < 0,163 \mid P_A = 0,2\right]$

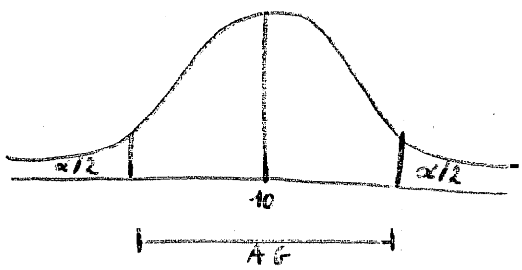
$$= P\left[Z < \frac{0,163 - 0,2}{\sqrt{\frac{0,2 \cdot 0,8}{60}}}\right] = P\left[Z < -0,716\right]$$

$= 0,2358$

→ $\boxed{1-\beta = 0,7642}$

⑧ $\mu = 10$ $\quad H_0: \mu = 10$

$\alpha = 0,1$ $\quad H_a: \mu \neq 10$



$$AG: \left[\mu_0 \pm t^{10-1}_{\alpha/2} \cdot \frac{s}{\sqrt{n}}\right]$$

$$= \left[10 \pm 1,833 \cdot \frac{3,5917}{\sqrt{10}}\right] = \left[7,92 ; 12,08\right]$$

$\bar{x} = 8,7 \in AG \Rightarrow H_0$: ✓

$s = \sqrt{s^2}$

$$s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n-1} = \frac{116,1}{9} = 12,9 \rightarrow s = 3,5917$$

⑨ a)

$H_0: \mu = 5{,}0$

$H_a: \mu \neq 5{,}0$

a) $\alpha = 0{,}05 \rightarrow \alpha/2 = 0{,}025 \rightsquigarrow z = 1{,}96$

$$AG = \left[\mu_0 \pm z_{\alpha/2} \cdot \frac{0{,}9}{\sqrt{40}} \right] = [4{,}72 ; 5{,}28]$$

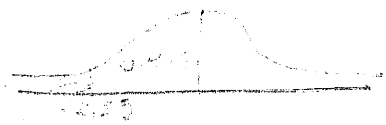
$\bar{x} = 5{,}5 \Rightarrow H_0$ verwerpen

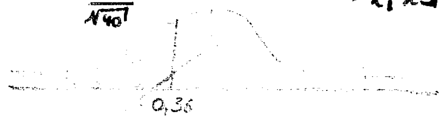
b) Onderscheidingsvermogen als $\mu = 5{,}6$

$1 - \beta$

$$\beta = P\left[\bar{x} \in AG_{H_0} \mid \bar{x} : N(5{,}6 ; 0{,}9)\right] = P[x < 5{,}28] = P\left[z < \frac{5{,}28 - 5{,}6}{\frac{0{,}9}{\sqrt{40}}}\right] = P[z < -0{,}356]$$

$-2{,}25$





$\rightarrow P[z < -2{,}25] = 0{,}0122 \Rightarrow 1 - \beta = 0{,}9878$

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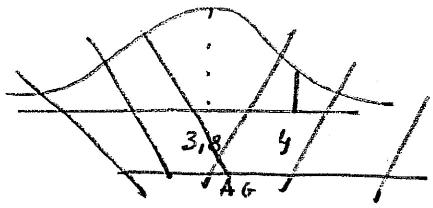
⑩ Globaal : 100 meisjes → 105 jongens

China : 262 → 338 (600)

(11) $n = 25$ $H_0 : \mu \geq 4$

a) $\bar{x} = 3,8$ $H_A : \mu < 4$

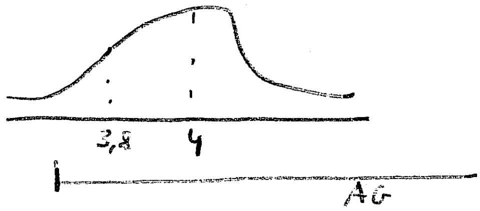
$s = 1,1$ $\alpha = 0,10$



~> moet t-waarde zijn

AG: $[\,4 - \cancel{\;\;}\,1,318 \cdot \frac{1,1}{\sqrt{25}}\, ; +\infty[\; = \;[3,72 ; +\infty[$

$\bar{x} = 3,8$ => H_0 n. verwerpen



b) 95% BBI ; $\mu < 0,2$; $s = 0,9$

$n = \frac{(1,96)^2 (0,9)^2}{(0,1)^2} = 311,17 \Rightarrow 312$ flessen

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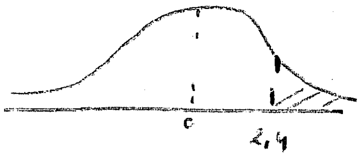
(12) a) $n = 36$, $\bar{x} = 38$, $s = 1,5$ $\boxed{\alpha = 0,01}$

$\alpha/2 = 0,005$



$H_0 : \mu \geq 41$

$H_A : \mu < 41$

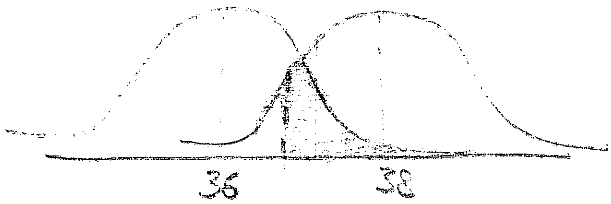
P-Waard $\frac{41 - 38}{\frac{1,5}{\sqrt{36}}} = 2,4$

$0,5 - 0,4918 = 0,0082 < 0,01$

=> H_0 verwerpen

b) Fout 2e soort , $\mu = 36$

$38 \pm 2,575 \cdot \frac{1,5}{\sqrt{36}} = [34,78 ; 41,22]$



$41 - Z_\alpha \cdot \frac{s}{\sqrt{n}}$

$\beta = P(\bar{x} \in AG \mid \mu = 36) = P(\bar{x} > 38,09)$

$= P\left(z > \frac{38,09 - 36}{1,5/6}\right)$

$= P(z > 1,67)$

$\beta = 0,0475$

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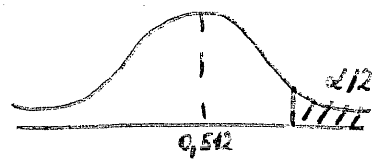
10 b)

$H_0: \quad P = 105/205 = 0{,}512$

$H_A: \quad P \neq 0{,}512$



$\hat{P} = \frac{338}{600} = 0{,}5633$

$\rightarrow \quad P\left[\hat{P} > 0{,}5633\right]$

$P\left[z > \frac{0{,}5633 - 0{,}512}{\sqrt{\frac{0{,}512 \cdot 0{,}488}{600}}}\right] = P\left[z > 2{,}5147\right]$

$= 0{,}006 < \alpha/2 = 0{,}025$

$\Rightarrow \cancel{H_0} \quad \Rightarrow$ Nein.

14
16

$\bar{x}_{2000} = 26{,}83$

$s^2_{2000} = \dfrac{8980 - \dfrac{103684}{\cancel{11}\,12}}{\cancel{12}\;11} = \dfrac{340}{11} = 30{,}91$

$\bar{x}_{2005} = 33$

$s^2_{2005} = \dfrac{13\,614 - \dfrac{156\,816}{12}}{11} = 49{,}64$

$$s^2_p = \frac{(n_{2000}-1)\cdot s^2_{2000} + (n_{2005}-1)\, s^2_{2005}}{n_{2000}+n_{2005}-2}$$

$$= \frac{11 \cdot 30{,}88 + 11 \cdot 49{,}64}{22} = \boxed{40{,}26}$$

<u>**BBI**</u> $\left[(\bar{x}_{2000} - \bar{x}_{2005}) \pm t^{\alpha/2}_{n_{2000}+n_{2005}-2} \cdot \sqrt{\dfrac{s^2_p}{n_{2000}} + \dfrac{s^2_p}{n_{2005}}} \right]$

$= (26{,}83 - 33) \pm t^{0{,}025}_{22} \cdot \sqrt{\dfrac{40{,}26}{12} + \dfrac{40{,}26}{12}}$

$= -6{,}17 \pm \overbrace{2{,}074 \cdot 2{,}59}^{5{,}37}$

$= [-11{,}54 \,;\, -0{,}80]$

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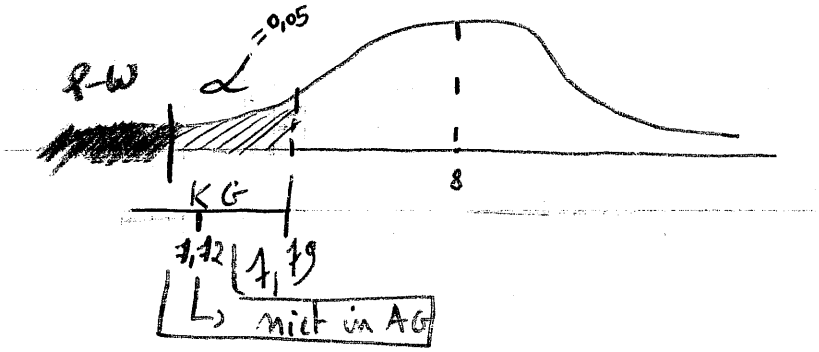
H8: Gej 6/5/8/10b

(6) a) H_A: $\mu < 8$ α H_0 (want bevat gelijkheid): $\mu \geq 8$

$\bar{x} = 7{,}72$

$\sigma = 0{,}5$



$AG = [\mu_0 - z_\alpha \frac{\sigma}{\sqrt{n}} \ , \ +\infty[$

σ geg? $\Rightarrow z$		
σ niet geg? $\Rightarrow$	$n \geq 30 \rightarrow z$ (CLS)	
	$n < 30 \rightarrow t$	

↳ $= [8 - 1{,}645 \cdot \frac{0{,}5}{4} \ , \ +\infty[\ = [7{,}79; +\infty[$

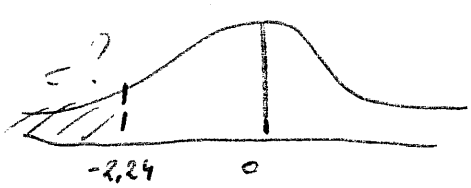
$\bar{X} = 7{,}72 \notin AG \rightarrow \cancel{H_0} \rightarrow H_a$ aannemen $\rightarrow$ Sokke is juist

b) $P[\bar{x} < 7{,}72]$

$= P\left[z < \frac{7{,}72 - 8}{\frac{0{,}5}{\sqrt{16}}}\right]$

$= P[z < -2{,}24]$

$= 0{,}0125$



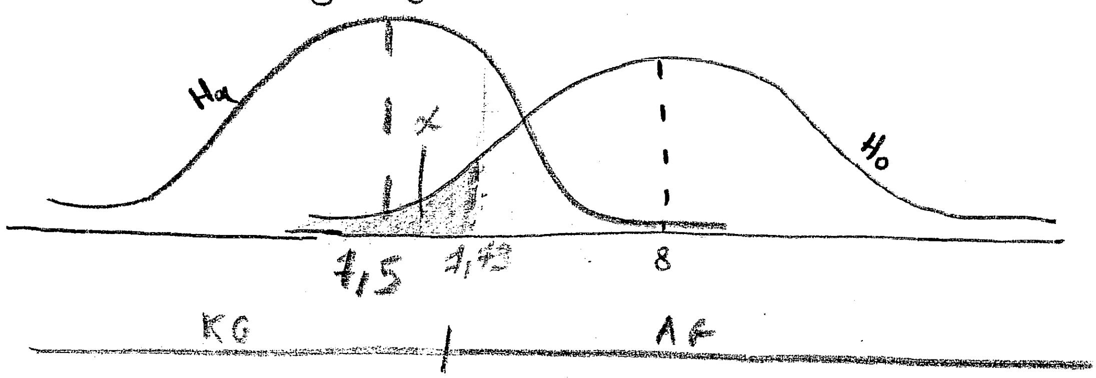
$= 0{,}5 - 0{,}4875$

$\boxed{< \ \alpha}$

↓

$\cancel{H_0} \rightarrow$ Sokke is juist

c) onderscheidingsvermogen : $1-\beta$



: Fout 1e soort

: " 2e "

$$\beta = P[\bar{x} \in AG_{H_0} \mid \mu_a = 1,5]$$

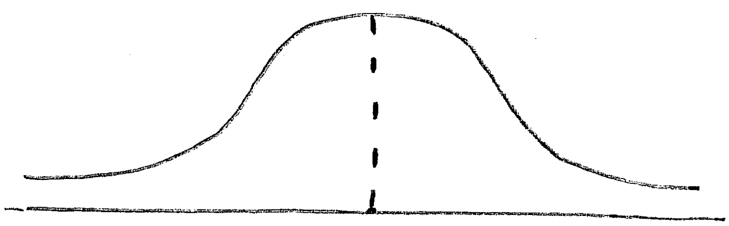
$$= P[\bar{x} > 1,19 \mid \mu_a = 1,5]$$

$$= P\left[z > \frac{1,19 - 1,15}{\frac{9,5}{4}}\right] = P[z > 2,32] = 0,0102$$

Bij geldigheid H_a 98,98% om de juiste beslissing te nemen bij geldigheid H_a.

(5) → FRACTIES → P (pop.fractie)

$\alpha = 0,05$



H_0 : $P \leq 0,1$

H_A : $P > 0,1$

$$AG =]-\infty \,;\, P_0 + z_\alpha \sqrt{\frac{P_0(1-P_0)}{n}}$$

$$=]-\infty \,;\, 0,1 + \overset{1,645}{\cancel{1,96}} \cdot \sqrt{\frac{0,1 \cdot 0,9}{60}}$$

$$=]-\infty, 0,163]$$

$$\hat{P} = \frac{8}{60} = 0,133 \in AG \rightarrow H_0 \Rightarrow$$

b)

$\beta = P[\hat{p} \in AG_{H_0} \mid p_A = 0{,}2]$

$= P[\hat{p} < 0{,}163 \mid p_A = 0{,}2]$

$= P\left[z < \frac{0{,}163 - 0{,}2}{\sqrt{\frac{0{,}2 \cdot 0{,}8}{60}}}\right] = P[z < -0{,}116] = 0{,}2358$

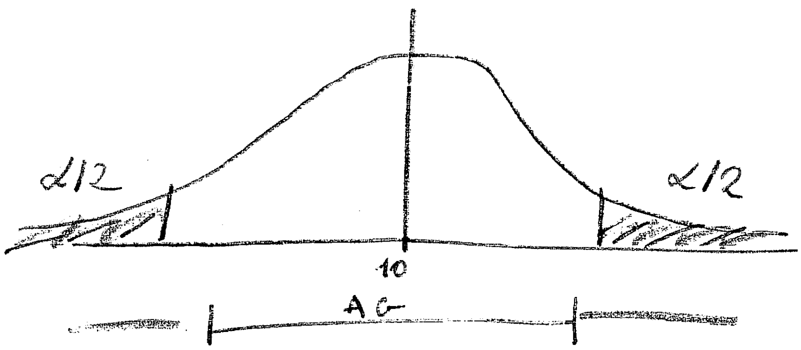
$\boxed{1 - \beta = 0{,}7642}$

σ geg $\rightarrow z$

σ n geg $\left\{ \begin{array}{l} GLS: z \\ t \end{array} \right.$

8. $\mu = 10$ $\quad H_0: \mu = 10$

$\alpha = 0{,}1$ $\quad H_a: \mu \neq 10$



AG $\left[\mu_0 \pm t^{10-1}_{\alpha/2} \cdot \frac{s}{\sqrt{n}}\right]$

$= \left[10 \pm \cancel{1{,}812}\ 1{,}833 \cdot \frac{3{,}5917}{\sqrt{10}}\right]$

$= [7{,}92 \,;\, 12{,}08]$

$\bar{x} = 8{,}7 \in AG \Rightarrow H_0$ ✓

$s = \sqrt{s^2}$

$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$

$= \frac{116{,}1}{9} = 12{,}9$

$\bar{x} = 8{,}7$

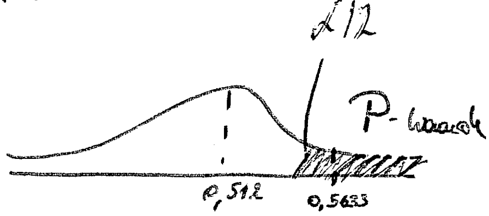
$\rightarrow s = 3{,}5917$

10b

100 m → 105 j
262 m → 338 j

$H_0: p = \frac{105}{205} = 0{,}512$

$H_A: p \neq 0{,}512$



$\hat{p} = \frac{338}{600} = 0{,}5633$

$= P[\hat{p} > 0{,}5633]$

$= P\left[Z > \frac{0{,}5633 - 0{,}512}{\sqrt{\frac{0{,}512 \cdot 0{,}488}{600}}}\right]$

$= P[Z > 2{,}5147]$

$= 0{,}006 \quad < \frac{\alpha}{2} = 0{,}025 \Rightarrow \cancel{H_0}$

$\Rightarrow$ NEE

19
~~20~~

(13) $n = 3456$

$H_0: \; p \geqslant 0{,}02 \qquad \alpha = 0{,}05$

$H_A: \; p < 0{,}2$

$$AG: \left[0{,}02 - 1{,}645 \cdot \sqrt{\frac{0{,}02 \cdot 0{,}98}{3456}} \; ; \; +\infty\right[$$

$$= \left[0{,}0161 \; ; \; +\infty\right[$$

$\hat{p} = \dfrac{62}{3456} = 0{,}018 \in A_G \Rightarrow H_0$ n̄ verwerpen.

Hoofdstuk 9

$$D_P^2 = \frac{(n_{2000}-1)\cdot s_{2000}^2 + (n_{2005}-1)\cdot s_{2005}^2}{n_{2000}+n_{2005}-2} = 40,26$$

(1) Onafh. steekproef, n. gepaard want n. noodz. zelfde winkels.

95% BBI → $\alpha = 0,05$

$$\text{BBI}: \left[(\bar{x}_{2000} - \bar{x}_{2005}) \overset{\text{w. tweezijdig}}{\pm} t^{\alpha/2}_{n_{2000}+n_{2005}-2} \cdot \sqrt{\frac{D_P^2}{n_{2000}} + \frac{D_P^2}{n_{2005}}} \right]$$

$s^2_{2000} = 30,88$ $s^2_{2005} = 49,64$

$\bar{x}_{2000} = 26,83$ $\bar{x}_{2005} = 33$

$$= (26,83 - 33) \pm \overset{2,074}{t^{0,025}_{22}} \cdot \sqrt{40,26\left(\frac{1}{12}+\frac{1}{12}\right)}$$

$= [-11,54 ; -0,80]$ → neg., dus in 2005 hogere waarde, toename v. 0,8 ~ 11,5%

$\bar{x}_D = 200$

s_D : 2262,5 − 2062,5

$= 130,93$

(2) → Gepaard.

$H_0 : \mu_{cca} - \mu_{na} \leq 0$

$H_A : \mu_{cca} - \mu_{na} > 0$

$$AG: \left] -\infty ; (\mu_D)_0 + t^{\alpha}_{n-1} \cdot \frac{s_D}{\sqrt{n}} \right]$$

$=]-\infty ; 87,12]$

→ $\bar{x} = 200 \notin AG$ ⇒ H_0 verwerpen

→ Advertering heeft effect

(11)

$$\boxed{\begin{array}{l} H_0 : P_j - P_o \geq 0 \\ H_A : P_j - P_o < 0 \end{array}}$$

of $H_0 : P_o - P_j \leq 0$

$H_A : P_o - P_j > 0$

$$AG: \left[(P_j - P_o)_0 - z_\alpha \cdot \sqrt{\frac{P_j(1-P_j)}{n_j} + \frac{P_o(1-P_o)}{n_o}} ; +\infty \right[$$

(0-hypothese)

$= [-0,14 ; +\infty[$

0 − 1,645 · 0,08

$$\hat{P} = \frac{n_j \cdot \hat{P}_j + n_o \cdot \hat{P}_o}{n_j + n_o} = 0,82$$

→ $\hat{P}_j = 25/32 = 0,78$

→ $\hat{P}_o = 46/55 = 0,84$

$\hat{P}_j - \hat{P}_o = -0,06 \in AG$

⇒ Gegevens geven geen aanleiding dat de rekenmachine toeneemt.

Hoofdstuk 9

③ a) Varianties

$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$ $\qquad \frac{10,2^2}{4,3^2} = 1,95 = F$

$H_A : \frac{\sigma_1^2}{\sigma_2^2} \neq 1$ $\qquad < F^{0,025}_{20,15} = 2,57$

$\frac{s_2^2}{s_1^2} \leq F^{\alpha/2}_{15,20} \longrightarrow H_0$ aanvaarden, want $1,95 < 2,57$

b) $H_0 : \mu_A = \mu_B \rightsquigarrow \mu_A - \mu_B = 0$

$H_A : \mu_A \neq \mu_B$

$$s_p^2 = \frac{(n_1 - 1) \cdot s_1^2 + (n_2 - 1) \cdot s_2^2}{n_1 + n_2 - 2} = \frac{20 \cdot 4,3^2 + 15 \cdot 10,2^2}{35} = 55,04$$

$$\Rightarrow (\mu_A - \mu_B)_{H_0} \pm t^{0,025}_{35} \cdot \sqrt{55,04\left(\frac{1}{21} + \frac{1}{16}\right)}$$
$$= 0 \pm 2,030 \cdot 2,88$$
$$= [-5,84 ; 5,84]$$

$\rightarrow 56 - 60,3 \in AG$

---

④ Grootst. grote steekproef

$H_0 : \mu_A \leq \mu_B \rightarrow \mu_A - \mu_B \leq 0$ $\qquad \alpha = 0,05$

$H_A : \mu_A > \mu_B$

$$AG \quad]-\infty ; 0 + 1,645 \cdot \sqrt{\frac{(1,95)^2}{45} + \frac{(2,0)^2}{65}}\,] =]-\infty ; 0,65]$$

$\bar{X}_A - \bar{X}_B = 0,9 \Rightarrow H_0$ verwerpen

⑤ a) 90%-BI

$$\left[42,7 \pm 1,645 \cdot \frac{5,2}{\sqrt{100}}\right] = \left[41,84 ; 43,56\right]$$

b) $\alpha = 0,05$

$$\bar{x}_A - \bar{x}_B \sim N\left(\mu_B - \mu_A , \sqrt{\frac{\sigma_A^2}{n_A} + \frac{\sigma_B^2}{n_B}}\right)$$

$H_0 : \mu_B - \mu_A \geq 0 \; : \; \mu_J \geq \mu_A$

$H_A : \mu_B - \mu_A < 0 \; : \; \mu_J < \mu_A$

AG $\left[0 - 1,645 \cdot \sqrt{\frac{5,2^2}{100} + \frac{4,5^2}{120}} \; ; \; +\infty\right[$

$= \left[-1,09 \; ; \; +\infty\right[$

$\mu_B - \mu_A = -1,6 \notin AG \Rightarrow H_0$ verwerpen

---

⑥ $H_0 : \mu_A - \mu_B \leq 0$ Waarom geen ½ ?

$H_A : \mu_A - \mu_B > 0$

$$\cancel{\left]-\infty \; ; \; 0 + 1,645 \cdot \sqrt{\frac{0,8^2}{21} + \frac{1,1^2}{21}}\right] = \left]-\infty \; ; 0,49\right]}$$

Variantieanalyse + t-score ! $\mu_A - \mu_B = 4,0 - 4,4 = -0,4 \in AG$

---

⑦ Gepaard, klein $\bar{x} = 2,5$ $s^2 = \frac{104 - 50}{7} = 7,71 \rightarrow s = 2,78$

$H_0 : \mu_D \leq 2$

$H_A : \mu_D > 2$ $\rightarrow \left]-\infty \; ; \; (\mu_D)_0 + t_7^{0,10} \cdot \frac{s_D}{\sqrt{n_D}}\right]$

$= \left]-\infty \; ; \; 2 + 1,39\right] = \left]-\infty \; ; \; 3,39\right]$

$\bar{x}_D = 2,5 \in AG \rightarrow H_0$ ✓ (Aanvaarden)

8) $\mu_A - \mu_{IT}$

$$(1387,1 - 1243,7) \pm 1,645 \sqrt{\frac{(337,5)^2}{160} + \frac{(252,8)^2}{120}}$$

$\rightarrow$ 90% BI = $143,4 \pm 58,04 \Rightarrow [85,36 \,;\, 201,44]$

9) $H_0: \mu_M - \mu_V = 2$

$H_A: \mu_M - \mu_V \neq 2$

$$\left[2 \pm t_{50}^{0,025} \cdot \sqrt{38,88\left(\frac{1}{21} + \frac{1}{31}\right)}\right] = [-1,55;\, 5,55]$$

$$s_p^2 = \frac{(n_1-1) \cdot s_1^2 + (n_2-1) \cdot s_2^2}{n_1+n_2-2} = 38,88$$

Erst Varianz!

$9,3 - 6,6 = 2,7 \in AG$

$\bar{X}_A = 3,5 \quad S_A = 0,3 \quad n_A = 21 \quad \bar{x}_B = 4,2 \quad s_B = 0,7 \quad n_B = 21 \quad \alpha = 0,05$

10) $\mu_A - \mu_B = 0 \quad \rightsquigarrow \quad H_0: \mu_A \geq \mu_B$

$\mu_A - \mu_B \neq 0 \qquad H_A: \mu_A < \mu_B$

Erst S lesen dann! (zu messen)

$$\left[0 - t_{40}^{0,025} \cdot \sqrt{s_p^2 \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)};\, +\infty\right[$$

$$\hookrightarrow s_p^2 = \frac{20 \cdot 0,3^2 + 20 \cdot 0,7^2}{40} = 0,29$$

$= [0 - 1,684 \cdot 0,17;\, +\infty[$

$= [-0,29\,;\, +\infty[\qquad \mu_A - \mu_B = 3,5 - 4,2 = -0,7 \notin AG$

$\hookrightarrow H_0$ Verwerfen.

Statistiek: af les 2

10.1

$H_0: \mu_A = \mu_B = \mu_C$

H_A: tenminste 2 μ's zijn $\neq$

$$SST = \sum_{i=1}^{p} n_i \cdot (\bar{x}_i - x)^2 = 256 + 294 + 20 = 570$$

$$F = \frac{MST}{MSE} = \frac{\frac{SST}{p-1}}{\frac{SSE}{n-p}}$$

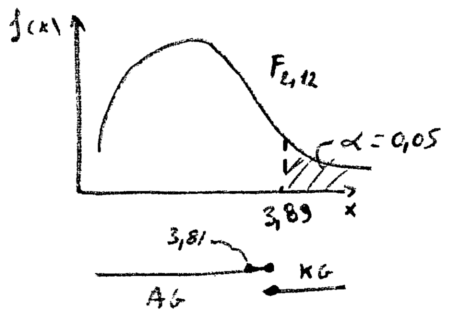
$$SSE = \sum_{i=1}^{p} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 = 196 + 310 + 392 = 898$$

$$= \frac{\frac{570}{3-1}}{\frac{898}{15-3}} = 3{,}81$$

→ Hoe groter, hoe groter kan dat je H_0 verwerpt

$$F^{\alpha}_{p-1, n-p} = F^{0,05}_{2, 12} = 3{,}89$$

$F < 3{,}89 \Rightarrow$ H_0 aanv. = de gemiddelde scores zijn gelijk / voorgaande ~~[illegible]~~ scholen zijn n. van belang



10.2 $n = 24$ $H_0 : \mu_A = \mu_B = \mu_C$

$\alpha = 0{,}05$ H_A : ten minste 2 μ's zijn $\neq$

TABEL

	Alleen EV		Weinig VV		Veel VV	
	x_{ij}	$(x_{ij} - \bar{x}_i)^2$	x_{ij}	$(x_{ij} - \bar{x}_i)^2$	x_{ij}	$(x_{ij} - \bar{x}_i)^2$
	3,8	6,40	8,6	9,12	20,6	21,07
	6,3	0,001	12,5	0,77	10,9	26,11
	7,6	1,61	5,0	43,82	17,4	1,93
	8,2	3,50	9,9	2,86	16,1	0,01
	8,1	3,13	16,3	21,90	23,0	48,86
	0,2	37,58	17,4	33,41	25,4	88,17
	4,9	2,05			4,0	144,24
	7,0	0,45			7,9	65,77
	10,9	20,88			18,8	7,78
n_i	9		6		9	
$\bar{x}_i$	6,33		11,62		16,01	
$\bar{x}$						11,28
$\Sigma(x_{ij} - \bar{x}_i)^2$		75,6		111,98		403,94
$(\bar{x}_i - x)^2$		24,50		0,12		22,37
$n_i \times (\text{''})$		220,5		0,72		201,33

$MST = \frac{SST}{p-1} =$ 211,28

$\Rightarrow F = 7{,}50 > F^{\alpha\, 0{,}05}_{2,\,21} = 3{,}47$

$MSE = \frac{SSE}{n-p} =$ 28,17

$\Rightarrow H_0$ verwerpen

10.4

TABEL

	S: x_{ij}	$(x_{ij}-\bar{x}_i)^2$	I: x_{ij}		V: x_{ij}		H: x_{ij}	
	2,1	0,01	1,9	0,09	2,1	0	2,8	0,04
	2,3	0,09	2,8	0,36	1,9	0,04	2,6	0
	1,6	0,16	1,6	0,36	2,0	0,01	2,1	0,25
	2,4	0,16	2,3	0,01	1,5	0,36	2,2	0,16
	2,2	0,04	2,4	0,04	2,7	0,36	3,0	0,16
	1,4	0,36	2,2	0	2,4	0,09	2,9	0,09
n_i	6		6		6		6	
$\bar{x}_i$	2		2,2		2,1		2,6	
$\bar{x}$								$\boxed{2,225}$

	S	I	V	H		
$\sum(x_{ij}-\bar{x}_i)^2$	0,82	0,86	0,86	0,7	→	3,24 ↓ SSE
$(\bar{x}_i-\bar{x})^2$	0,0506	0,0001	0,016	0,141		
$n_i \cdot (\bar{x}_i-\bar{x})^2$	0,3036	0,001	0,096	0,846	→	1,247 ↓ SST

$$\rightarrow MST = \frac{SST}{p-1} = 0,416$$

$$\rightarrow MSE = \frac{SSE}{n-p} = 0,162$$

$$\Rightarrow F = \frac{MST}{MSE} = 2,57$$

$$2,57 < F^{\alpha}_{(p-1),(n-p)} = 3,10 \Rightarrow H_0 \text{ n. verworpen.}$$

$$\| \; F^{0,05}_{3,20}$$

$P_{11} = P_{R1} \cdot P_{C1}$

11.1 $n = 400$ $\alpha = 0{,}05$

H_0: Verand. zijn onafh.

H_A: " afh.

M:	0,25	0,125	0,09375	0,15625
V:	0,15	0,075	0,05625	0,09375

$\rightarrow E(n_{ij}) = n \cdot p_{ij} \rightarrow$

$\hat{E}$

100	50	37,5	62,5
60	30	22,5	37,5

$$\chi^2 = \sum \frac{(n_{ij} - \hat{E}(n_{ij}))^2}{\hat{E}(n_{ij})} =$$

4,84	0,32	1,127	6,084
8,07	0,53	1,88	10,14

$= 32{,}991 >$ ..

$\rightarrow H_0$ verwerpen

$\chi^{2, 0{,}05}_{3} = 7{,}81$

10.4

11.2

$\alpha = 0{,}05$

onder de o-hyp

H_0 : $P_{1,0} = 0{,}3$; .... ; $P_{5,0} = 0{,}05$

H_A : andere verdeling

$$\chi^2 = \frac{\sum_{i=1}^{k} (n_i - E(n_i))^2}{E(n_i)}$$

Klassen	n_i	$P_{i,0}$	$E(n_i)$
0-5	2	0,3	$\times 20 \rightarrow$ 6
5-10	3	0,3	6
10-15	5	0,25	5
15-20	1	0,1	2
≥ 20	3	0,05	1
	$n=20$	1	

7; 10 — 15 — 0,4 — 8 $\Rightarrow E(n_i) < 5$

Werkhypothesen

bv. $E(n_i) > 5$

$$\chi^2 = \sum_{i=1}^{k} \frac{(n_i - E(n_i))^2}{E(n_i)} = \frac{(2-6)^2}{6} + \frac{(3-6)^6}{6} + \frac{(15-8)^2}{8}$$

$= 10{,}29$

$\chi^2 = 10{,}29 \overset{?}{\lessgtr} \chi^{2,\alpha} = \chi_2^{2;0{,}05} = 5{,}99$

$\Rightarrow$ Ho verwerpen $\rightsquigarrow$ Studenten hebben andere gestaangeraag dan V.

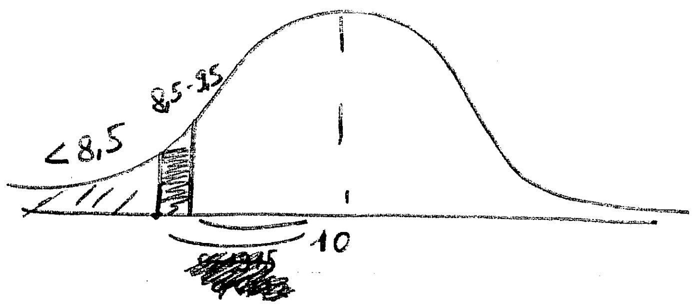
11.1 - 10.4 - 11.3

11.3) H_0 : X volgt $N(10,1)$ $\qquad$ $n = 250$

H_A: X volgt een andere verdeling

$P_{1,0} \rightarrow P[X<8,5 \mid X \sim N(10,1)]$
$= P\left[z < \frac{8,5-10}{1}\right] = P[z<-1,5]$
$= 0,0668$

Klassen	n_i	$p_{i,0}$	$E(n_i)$
<8,5	30	0,0668	16,7
8,5 - 9,5	60	0,2417	60,425
9,5 - 10,5	85	0,383	95,75
10,5 - 11,5	50	0,2417	60,425
>11,5	25	0,0668	16,7



→ χ^2-test

$$\chi^2 = \sum \frac{(n_{ij} - E(n_{ij}))^2}{E(n_{ij})} = 10,59 + 0,003 + 1,21 + 1,80 + 4,13$$
$$= 17,73$$

→ vgl. met $\chi^{2,\,0,05}_{4}$ $\qquad$ > 9,49 → H_0 verwerpen.

10.3

$SST = 12564 \rightarrow MST = \frac{12564}{5} = 2512,8$

$S\cancel{M}SE = 24992 \rightarrow MSE = \frac{24992}{30} = 833,07$

$F = \frac{2512,08}{833,07} = 3,02 < F^{0,01}_{5,30} = 3,70 \rightarrow H_0$ aanvaarden

---

11.4

$n = 200$

$\alpha = 0,1$

			$\hat{E}(n_{ij})$
A	34%	71/200 = 35,5%	68
B	23%	40/200 = 20%	46
C	15%	23/200 = 11,5%	30
D	28%	66/200 = 33%	56

$$\chi^2 = \sum \frac{(n_{ij} - \hat{E}(n_{ij}))^2}{\hat{E}(n_{ij})} = 0,13 + 0,78 + 1,63 + 1,79 = 4,33$$

$$\chi^{2;0,1}_{3} = \cancel{11,35}\ 6,25 \Rightarrow H_0 \text{ n. verwerpen}$$

11.5 $\alpha = 0,05$ $n = 500$

18	16	16
57,6	51,2	51,2
104,4	92,8	92,8

Fitheid	Geen	Weinig	Veel
zwak	0,04	0,03	0,03
matig	0,12	0,10	0,10
goed	0,20	0,18	0,18
zeer goed	0,01	0,01	0,01

samen

$\rightarrow \hat{E}$

$$\chi^2 = \sum \frac{(n_{ij} - \hat{E}(n_{ij}))^2}{\hat{E}(n_{ij})} = \cancel{20,24} \ 0,05 + 0,07 + 0,07 + 1,35 + 0,72 + 4,5 + 0,24 + 0,26 + 2,37$$

$$0,0556 + 0,0625 + 0,7563 + 1,0125 + 3,720 + 0,300 + 0,559 + 1,766$$

$$= 8,23 \rightarrow \chi^{2;0,05}_{4} < 9,49 \Rightarrow H_0 \text{ n. verwerpen}$$

H12

1) SSE en s:

a) $n = 20$, $SS_{yy} = 95$, $SS_{xy} = 50$, $\hat{\beta}_1 = 0,75$

$\hookrightarrow SSE = SS_{yy} - \hat{\beta}_1 \cdot SS_{xy} = 95 - 0,75 \cdot 50 = \boxed{57,5}$ ✓

$s = \sqrt{\frac{57,5}{18}} = \boxed{1,79}$ ✓

b) $n = 40$, $\Sigma y_i^2 = 860$, $\Sigma y_i = 50$, $SS_{xy} = 2700$, $\hat{\beta}_1 = 0,075$

$\hookrightarrow SSE = SS_{yy} - \hat{\beta}_1 \cdot SS_{xy} \Rightarrow 797,5 - 0,075 \cdot 2700 = \boxed{595}$ ✓

$\hookrightarrow 860 - \frac{50^2}{40} = 797,5$

$\hookrightarrow s = \sqrt{\frac{595}{38}} = \boxed{3,96}$ ✓

2) Pearson: $r = \frac{SS_{xy}}{\sqrt{SS_{xx} \cdot SS_{yy}}} \rightarrow SS_{xy} = \Sigma x_i y_i - \frac{\Sigma x_i \cdot \Sigma y_i}{n}$

$= 22.600,5 - \frac{226536,4}{10} = -53,14$

$SS_{xx} = \Sigma(x_i^2) - \frac{(\Sigma x_i)^2}{n} = 45.196 - \frac{43.824,4}{\cancel{n}} = 1371,6$

$SS_{yy} = 12.991,44 - 11.710,08 = 1281,36$

$\Rightarrow r = \frac{-53,14}{\sqrt{1371,6 \cdot 1281,36}} = -0,04$ ✓

$$\hat{B}_1 = \frac{SS_{xy}}{SS_{xx}} \to SS_{xy} = \left[\Sigma(x_i y_i) - \frac{\Sigma x_i \cdot \Sigma y_i}{n}\right] = 912,1 - 871,30 = 40,8$$

$$\to SS_{xx} = \Sigma(x_i^2) - \frac{\Sigma(x_i)^2}{n} = 36,76 - 27,84 = 8,92$$

3) a) $\hat{y} = \hat{B}_0 + \hat{B}_1 \cdot x$

$$\to \hat{B}_0 = \bar{y} - \hat{B}_1 \cdot \bar{x} = 372,68$$

$$\Rightarrow \hat{B}_1 = \frac{40,8}{8,86} = 4,60$$

$$\hat{y} = 53,24 + 4,60 \cdot x$$

b) $s = \sqrt{\frac{SSE}{n-2}} \to SSE = SS_{yy} - \hat{B}_1 \cdot SS_{xy}$

$$SS_{yy} = 27.853,49 - \frac{27.268,8}{\ldots} = 584,69$$

$$\Rightarrow \sqrt{\frac{584,69 - 4,60 \cdot 40,8}{5}} = 8,91$$

c) 90% BBI

$$4,60 \pm t_5^{0,05} \cdot \frac{s}{\sqrt{SS_{xx}}} = 4,60 \pm \overbrace{2,015}^{2,015} \cdot \frac{8,91}{\sqrt{8,86}}$$

$$\overbrace{}^{6,03}$$

$$Bi_{90\%} = [-1,43 ; 10,63]$$ ✓

~~d) Betrouwbaarheid : $\alpha = 0,05$~~

~~$H_0 : \hat{B}_1 = 0$~~

~~$H_A : \hat{B}_1 \neq 0$~~

~~$\Rightarrow \frac{\hat{B}_1}{\frac{s}{\sqrt{SS_{xx}}}} = 0,11$~~

d) $r = \frac{SS_{xy}}{\sqrt{SS_{xx} \cdot SS_{yy}}} = \frac{40,8}{\sqrt{8,86 \cdot 584,69}} = 56,67\%$

$= 31,93\%$

?

$$r^2 = 1 - \frac{\overbrace{SSE}^{SS_{yy}}}{SS_{yy}} = \frac{SS_{yy} - \hat{B}_1 \cdot SS_{xy}}{SS_{yy}}$$

$$1 - \frac{SS_{yy} + B_1 \cdot SS_{xy}}{SS_{yy}} = 1 - \frac{4,58 \cdot 40,8}{584,69}$$

$$= \cancel{1-1} \; \frac{\hat{B}_1 \cdot SS_{xy}}{SS_{yy}} = 31,95$$

Gefening les stat 2 : H12

4) a) Gvelast (+) of mobiliteit (-) ?

↳ Dichter = goedkoper ↳ $\overset{\text{dichter}}{\dots}$ = duurder

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 \cdot x$$

$$5829 - \frac{11{,}3 \cdot 1705}{5} = -10{,}3$$

$$\hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} \longrightarrow SS_{xy} = \sum x_i y_i - \frac{\sum x_i \cdot \sum y_i}{n}$$

Altijd pos. $SS_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = 67{,}57 - \frac{(11{,}3)^2}{5} = 7{,}71$

y_i	x_i	$x_i \cdot y_i$	x_i^2	y_i^2
325	5,4	1755	29,16	105 625
360	4,2	1512	17,64	129 600
300	3,2	960	10,24	90 000
380	1,8	684	3,24	144 400
340	2,7	918	7,29	115 600
$\sum y_i = 1705$	$\sum x_i = 11{,}3$	$\sum x_i y_i = 5829$	$\sum x_i^2 = 67{,}57$	$\sum = 585225$
$\bar{y} = 341$	$\bar{x} = 3{,}46$			

$\hat{\beta}_1 = \frac{-10{,}3}{7{,}71} = -9{,}12$ → neg. verband; hoe hoger x, hoe lager y

→ argument mobiliteit

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x} = 341 - (-9{,}12) \cdot 3{,}46 = 372{,}56$$

$$\Rightarrow \hat{y} = 372{,}56 - 9{,}12\,x$$

↳ $\hat{\beta}_0$ → Snijpunt met de x-as, echter ⇒ p = 372 560 ~~km~~ euro

⇝ kunnen we niet, zeggen met dit model, want geen w. in dit bereik

⇝ Per km $\overset{\text{extra}}{\text{verwijderd}}$ v/d autostrade, € 9120 minder w.

b) $SSE = \Sigma (y_i - \hat{y}_i)^2$

$$= \underbrace{SS_{yy}} - \hat{B}_1 \cdot SS_{xy} = 3820 - (-9{,}12) \cdot (-70{,}3)$$

$$= 3178{,}86$$

$$SS_{yy} = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$= 585225 - \frac{(1705)^2}{5} = 3820$$

---

s^2 (=schatting variantie model) $= \dfrac{SSE}{n-2} = \dfrac{3178{,}86}{3} = 1059{,}62$

$\to s = 32{,}55$

$\pm 32{,}55$

68% v/d waarden binnen ~~[illegible]~~
v/d schatting v/h model
(x-waarde invullen)

---

c) Determinatie coëfficiënt

stuk dat je $\bar{n}$ · kan verklaren

$$R^2 = \frac{SS_{yy} - SSE}{SS_{yy}} = 1 - \frac{SSE}{SS_{yy}} = 1 - \frac{3178{,}56}{3820}$$

$= 0{,}168$ v. var. door het model

→ 83% door iets anders!

↳ Hoe dichter bij 0, hoe slechter het model

---

d) $\hat{y}\,(x=4) \Rightarrow 372{,}56 - 9{,}12 \cdot 4 = 336{,}08 \to 336\,080$ euro

---

e) $r = \dfrac{SS_{xy}}{\sqrt{\underset{(+)}{SS_{xx}} \cdot \underset{(+)}{SS_{yy}}}} = \dfrac{-70{,}3}{\sqrt{7{,}72 \cdot 3820}} = -0{,}410$

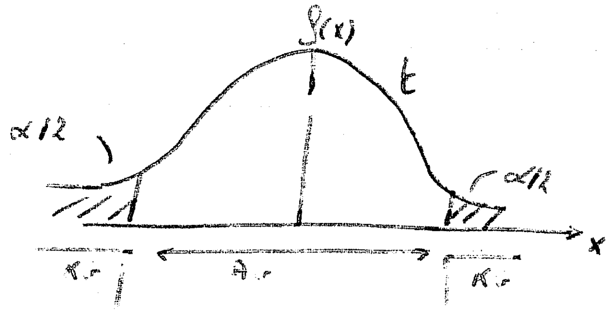
$$\hat{y} = 372{,}56 \underbrace{-9{,}12}_{\hat{B}_0} X$$

f) $H_0 : \hat{B}_1 = 0$ ⇝ Geen invloed v. x op y

$H_A : \hat{B}_1 \neq 0$

0 onder H_0

$$t = \frac{\hat{B}_1 - \hat{B}_{1,0}}{s\hat{B}_1 = \frac{s}{\sqrt{SS_{xx}}}} = \frac{-9{,}12}{\frac{32{,}55}{\sqrt{7{,}71}}} = -0{,}777$$



$$t_{n-2}^{\alpha/2} = t_3^{0{,}025} = 2{,}353$$

$-2{,}353$

$$\boxed{-0{,}777 \in A.G.}$$

→ door symm. verdeling mag je $|t|$ nemen + rechterkant t-verdeling

$$|t| = 0{,}777 < t_{n-2}^{\alpha/2} = t_3^{0{,}025} = 2{,}353 \Rightarrow H_0 \text{ aanv.}$$

x verklaart y n. voldoende op sign. niveau

→ enkel tweezijdige toets kennen (=, ≠)

12.5 & 12.6

a) Lijn v/d kleinste kwadraten

$$\hat{y} = \hat{B}_0 + \hat{B}_1 . x$$

$$\hat{B}_1 = \frac{SS_{xy}}{SS_{xx}}$$

$$SS_{xy} \rightarrow \sum x_i . y_i - \frac{\sum x_i . \sum y_i}{n} = 6123 - 5852 = 271$$

$$SS_{xx} \rightarrow \sum x_i^2 - \frac{(\sum x_i)^2}{n} = 3448 - 3388 = 60$$

$$\Rightarrow \hat{B}_1 = 4,51667$$

$$\hat{B}_0 = \bar{y} - \hat{B}_1 . \bar{x} = 38 - 4,52 . 22 = -61,44$$

$$\Rightarrow \boxed{\hat{y} = -61,44 + 4,52x}$$

b) $s^2 = \frac{SSE}{n-2} \rightarrow SSE = SS_{yy} - \hat{B}_1 . SS_{xy}$

$$SS_{yy} \rightarrow \sum y_i^2 - \frac{(\sum y_i)^2}{n} = 11452 - 10108 = 1344$$

~~[illegible]~~ ~~[illegible]~~

$$1344 - 4,52 . 271 = 119,08$$

$$\rightarrow s^2 = \frac{119,08}{5} \rightarrow s = 4,88$$

c) Bruikbaarheid model

$H_0 : B_1 = 0$

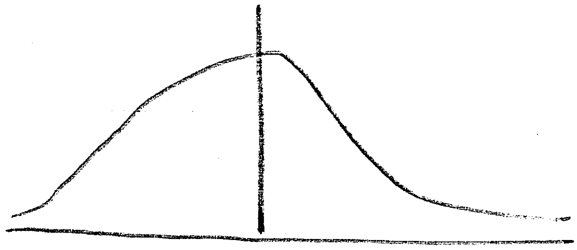
$H_A : B_1 \neq 0$

$$\frac{\hat{B}_1 - B_{1,0}}{\left(\frac{s}{\sqrt{SS_{xx}}}\right)} = 7,17$$

$$\rightarrow t_5^{0,05} = 2,015$$

$$7,17 > 2,015 \Rightarrow H_0 \text{ verwerpen}$$

d) 90% BBI voor B_1



$$\hat{B}_1 \pm t \cdot s_{\hat{B}_1}$$

$$s_{\hat{B}_1} = \frac{s}{\sqrt{SS_{xx}}} = \frac{4,85}{\sqrt{60}} = 0,63$$

$$1,52 \pm t_5^{0,05} \cdot 0,63$$
$$= 1,52 \pm 1,27 \Rightarrow [0,25 ; 2,79] \checkmark$$

BBI	Pop. parameter	Schatter		verdeling	St. schatter
	B_1	$\hat{B}_1$	$\pm$	$t_{n-2} \times$	$s_{\hat{B}_1} = \frac{s}{\sqrt{SS_{xx}}}$
Kleine steekproef	μ	$\bar{x}$		$t_{n-1} \times$	$s_{\bar{x}} = \frac{s}{\sqrt{n}}$

e) $x = 25$: $VI_{90\%} = \hat{y} + t_{n-2}^{\alpha/2} \cdot s \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_{xx}}}$

$$\cancel{\hat{y} = 61,51 + 1,52 \cdot 25} = 51,56 + 2,015 \cdot 4,85 \sqrt{1 + \frac{1}{7} + \frac{(25-22)^2}{60}}$$
$$= 51,56$$

12.6

a) Pearson:

$$r = \frac{SS_{xy}}{\sqrt{SS_{xx} \cdot SS_{yy}}} = \frac{250}{\sqrt{850 \cdot 230}} = 0,56$$

b) Positief verb. tussen x & y ⇒ (pos. correlatie) ⇒ stijgend

c) Regressielijn:

$$\hat{y} = \hat{B}_0 + \hat{B}_1 x$$

$$\hat{B}_1 = \frac{SS_{xy}}{SS_{xx}} = 0,2941$$

$$\hat{B}_0 = \bar{y} - \hat{B}_1 \cdot \bar{x} \Rightarrow \boxed{\hat{y} = -10,59 + 0,294x}$$

d) $SSE = SS_{yy} - \hat{B}_1 \cdot SS_{xy} = 156,5$

$$s^2 = \frac{SSE}{n-2} = 1,60$$

e) $\hat{y}(x = 190)$

$$-10,59 + 0,294 \cdot 190 \Rightarrow 45,27$$

7) $n = 10$

a) $\hat{y} = \hat{B}_0 + \hat{B}_1 . x$

$\hat{B}_1 = \frac{SS_{xy}}{SS_{xx}}$, $SS_{xy} = 40,92$, $SS_{xx} = 130$ $\Rightarrow 0,32$

$\hat{B}_0 = \bar{y} - \hat{B}_1 . \bar{x}$

$= 2,8 - 0,32 . 6 = \cancel{\ldots}\ 0,88$

$\rightarrow \hat{y} = 0,88 + 0,32 . x$

b) $\cancel{\hat{y}} = \quad \in 1260$

c) 99% CI for B_1

$s = \sqrt{\frac{SSE}{n-2}} = \cancel{0,18}$, $SSE = SS_{yy} - \hat{B}_1 . SS_{xy} = 13,39 - 0,32 . 40,92 = \cancel{0,25}$

$0,32 \pm t_8^{0,005} . \frac{s}{\sqrt{SS_{xx}}}$

$SS_{xx} = 130$

$t_8^{0,005} = 3,355$

$\boxed{\hat{y} = 0,911 + 0,315x}$

$= 0,32 \pm 3,355 . 0,02 = [0,24 ; 0,39]$

d) $PI_{90\%} = \left[\hat{y} \pm t_8^{0,05} . \cancel{s}\sqrt{1 + \frac{1}{10} . \frac{(5-6)^2}{130}} \right]$

$1 + 0,1 + 0,0077$

$= 2,48 \pm 1,86 . 0,18 . \sqrt{1 + \frac{1}{10} + \frac{1}{130}}$ (0,1768)

$= 2,48 \pm 0,33 . 1,052$

$= [2,13 ; 2,82]$

$[2,02 ; 2,96]$

Algemene opgaven

1) $P(E)$ $\quad \boxed{\begin{array}{l} H_0: \text{onafh} \\ H_A: \text{afh} \end{array}}$ $\quad \alpha = 0,01$

0,01	0,02	0,03	0,04	→	5	10	15	20
0,03	0,06	0,09	0,12		15	30	45	60
0,03	0,06	"	"		"	"	"	"
0,03	0,06	"	"		"	"	"	"

$$\rightarrow \chi^2 = \sum \left(\frac{(n_{ij} - \hat{E}(n_{ij}))^2}{\hat{E}(n_{ij})} \right) = 5 + 0 + 1,67 + 0 + 1,67 + 3,33 + 0,56 + 1,67 + 0,27 + 0,83 + 5 + 1,07 + 0,27 + 0,83 + 5 + 5,4 = 32,57$$

$$\chi^2 = 32,57 \overset{?}{>} \chi^{2,\,0,01}_{9} \quad (21,66) \Rightarrow \boxed{H_0 \text{ verwerpen}}$$

✓

---

2) $\bar{X} = 6,83$

	Thuis x_{ij}	$(x_{ij} - \bar{x}_i)^2$	KOT x_{ij}		SH x_{ij}	
	7,8	0,16	8,2	1,44	5,8	0,09
	6,3	1,21	6,1	0,81	6,6	0,25
	7,7	0,09	5,9	1,21	8,0	3,61
	5,8	2,56	7,3	0,09	5,8	0,09
	8,5	1,21	6,6	0,16	4,6	2,25
	8,2	0,64	6,9	0,01	6,4	0,09
	6,9	0,25	7,8	0,64	6,2	0,01
	8,0	0,36	7,2	0,04	5,4	0,49
n_i	8	6,81	8		8	
$\bar{x}_i$	7,4		7	4,4	6,1	6,88
$\sum(x_{ij} - \bar{x}_i)^2$						
$(\bar{x}_i - \bar{x})^2$	0,33		~~0,11~~ 0,03		0,53	
$n_i \times$ "	2,64		0,24		4,24	

$H_0 = \bar{m}.$ afh
$H_A =$ afh.

$\cdot SST = \sum_{i=1}^{p} \left(n_i \cdot (\bar{x}_i - \bar{x})^2\right) = 7,12$

$\hookrightarrow MST = \frac{7,12}{2} = 3,56$

$F = \frac{3,56}{0,86} = 4,14$

$\cdot SSE = \sum_{1}^{p} \sum_{1}^{n_i} (x_{ij} - x_i)^2 = 18,09$

$\hookrightarrow$ ~~SS~~ $MSE = \frac{18,09}{21} = 0,86$

$F_{2,21}^{0,05} = 3,47$

$4,14 > 3,47 \Rightarrow H_0$ verwerpen

---

(3) $H_0 : \mu_1 \geq \mu_2 \rightarrow \mu_1 - \mu_2 \geq 0$
$H_A : \mu_1 < \mu_2$

$n = 20$

$y_{01} = 164,6$, $s_{01} = 17,2$
$y_{02} = 143,2$, $s_{02} = 15,9$

) eerst variantie-toets!

$$s_p^2 = \frac{(n_1 - 1) \cdot s_1^2 + (n_2 - 1) \cdot s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{19 \cdot 17,2^2 + 19 \cdot 15,9^2}{38}$$

$$= 274,33$$

$$\left]-\infty ; 0 \pm t_{38}^{0,05} \cdot \sqrt{\frac{s_p^2}{n_A} + \frac{s_p^2}{n_B}}\right]$$

$$\left]-\infty ; 0 + 1,686 \cdot \sqrt{27,48}\right]$$

[illegible]

$164,6 - 143,2 = 21,4 \notin A_G$

OPL $]-\infty ; 8,82]$ $\rightarrow H_0$ verwerpen

[illegible]

$$\frac{19 \cdot 295,84 + 19 \cdot 252,81}{38}$$

$$\frac{5620,96 + 4803,39}{38} = 274,33$$

4) a)

$n = 200$ $\quad\quad \alpha = 0{,}05 \quad\quad H_0: \ p \geq 0{,}25$

$\hat{p} = \frac{38}{200} = 19\% \quad\quad H_A: \ p < 0{,}25$

$\cancel{0{,}19 \pm 1{,}96 \cdot \sqrt{\frac{0{,}19 \cdot 0{,}81}{200}}}$

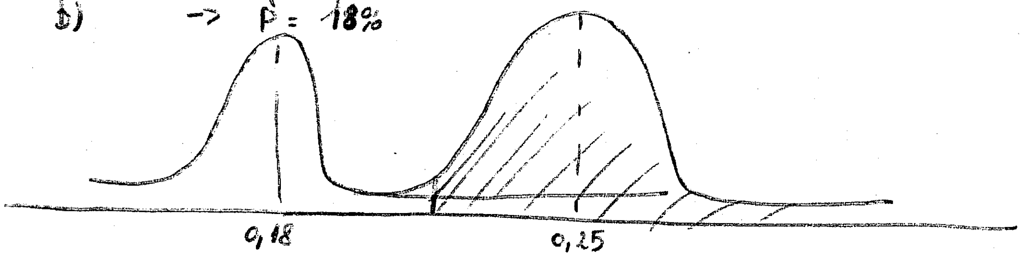
$$AG = \left[\, 0{,}25 - \overset{1{,}645}{\cancel{1{,}96}} \cdot \sqrt{\frac{p_0 \cdot (1-p_0)}{n}} \; ; \, +\infty \right[$$

$$= \left[\cancel{0{,}19 ; +\infty}\right[\quad\quad [0{,}20 \, ; +\infty[$$

$\hat{p} = \frac{38}{200} = 19\% \quad \notin AG \quad\quad \alpha = 0{,}05$

fout 2e soort: Kans dat H_0 w. geaccepteerd terwijl H_0 ongeldig is

b) $\rightarrow \hat{p} = 18\%$



$n = 200$

$P[p > 0{,}20 \mid \mu = 0{,}18] \rightarrow 0{,}18$

$$z = \frac{0{,}20 - 0{,}18}{\sqrt{\frac{0{,}18 \cdot (1 - 0{,}18)}{200}}} \checkmark \quad\quad \boxed{= \sqrt{\frac{0{,}1476}{200}} = 0{,}0272}$$

$$= 0{,}74$$

$P[z > 0{,}74] = 0{,}23$

5) a) $n = 16$

$$\boxed{\bar{x} - t_{15}^{\alpha/2} \cdot \frac{S}{\sqrt{n}} < \mu < \bar{x} + \dots}$$

2,131

$$115,2 \pm t_{15}^{0,025} \cdot \frac{8}{4}$$

$$= 115,2 \pm 2,131 \cdot 2 \quad \Rightarrow \quad [110,94 \; ; \; 119,46]$$

---

b) $\mu = 110$ $\qquad \mu = 115,2$

$\sigma = 16$

$$\boxed{\begin{array}{l} H_0 : \mu = 110 \\ H_A : \mu \neq 110 \end{array}}$$

$$Z = \frac{110 - 115,2}{\frac{16}{4}} \rightarrow 2,08 = 0,4812$$

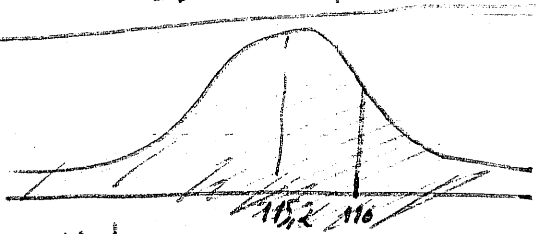
$\rightarrow$ p-waarde $= 0,0188 < 0,025$

$$\boxed{P(X > 115,2) = 0,0188 < \frac{\alpha}{2} = 0,025}$$

$\Rightarrow$ H_0 verwerpen

! c) $\mu = 116$

$\rightarrow$ onderscheidingsvermogen



$$AG = [105,1 \; ; \; 114,9]$$

$$\beta = P(105,1 < x < 114,9 \mid \mu_A = 116)$$

$$= 0,33 - 0 \quad \Rightarrow \quad 1 - \beta = 0,67$$

---

d) $n_B = 11$ $\qquad \sigma_B = 6,2$ $\qquad \alpha = 0,05$

$n_M = 7$ $\qquad \sigma_M = 8,8$

$$\boxed{\frac{S^2_{ma}}{S^2_{ba}}} \qquad \boxed{S, \text{ niet } \sigma}$$

$H_0 : \frac{\sigma_B^2}{\sigma_M^2} = 1$ $\qquad H_A : \frac{\sigma_B^2}{\sigma_M^2} \neq 1$

$$\frac{8,8^2}{6,2^2} = 2,015 < 4,07 \qquad F_{6,10}^{\alpha/2} = 4,07$$

$\Rightarrow$ H_0 n. verwerpen

c) $\mu_B = 132$ $\mu_M = 137$ $\alpha = 0{,}05$

$H_0: \cancel{\mu_B = \mu_M}$ $\mu_B - \mu_M = 0$

$H_A: \mu_B \neq \mu_M$ $\mu_B - \mu_M \neq 0$

~~$t_{0,025}$ ... $2{,}447$~~

STANDAARDAFW.

$\boxed{S_p^2 = 53{,}07}$

$0 \pm 2{,}12 \cdot \sqrt{\frac{6{,}2^2}{11} + \frac{8{,}8^2}{7}}$

$[(\mu_M - \mu_B)_0 - t^{\alpha/2}_{..}\sqrt{\frac{1}{n_M} + \frac{1}{n_B}}$ [redacted]

$0 \pm 1{,}96 \cdot \sqrt{\ldots}$

$\Rightarrow -5 \in [-7{,}47;\ 7{,}47] \Rightarrow H_0$ aanv.

OPL: $\left[(\mu_M - \mu_B)_0 - t^{\alpha/2}_{n_M+n_B-2}\sqrt{\left(\frac{1}{n_M} + \frac{1}{n_B}\right) S_p^2}\ ;\ (\mu_M - \mu_B)_0 + t \ldots\right.$

---

6) $H_0: \mu_V \geq \mu_M \Rightarrow \mu_V - \mu_M \geq 0$

$H_A: \mu_V < \mu_M$ $\mu_V - \mu_M < 0$

$\alpha = 0{,}05$

AG $\left[0 - 1{,}645 \cdot \sqrt{\frac{1{,}01^2}{1116} + \frac{1{,}10^2}{1171}}\ ;\ +\infty\right[$

$= [-0{,}072\ ;\ +\infty[$

$\cancel{0{,}22}$ $\mu_V - \mu_M = 0{,}22 \in AG$

??

OPL $H_0: \mu_V \leq \mu_M \rightarrow \mu_V - \mu_M \leq 0$

$H_A: \mu_V > \mu_M$

$\rightarrow AG: \left]-\infty\ ;\ 0 + 1{,}645 \cdot \sqrt{\ldots}\right]$

$=]-\infty\ ;\ 0{,}072]$

$0{,}22 \notin AG \Rightarrow H_0$ verwerpen

4T

(4) $n = 2344$ $\alpha = 0{,}05$

0, 10 0, 148 0, 258 → $\hat{E}$

0, 10 0, 14 0, 25

234	347	605
234	328	586

$4{,}11 + 1{,}66 + 5 + 5{,}24 + 0{,}60 + 5{,}94 = 22{,}55$ >

$\cancel{E_2^{0,050}}$ $\chi_2^{0,05} = 5{,}991$

↓

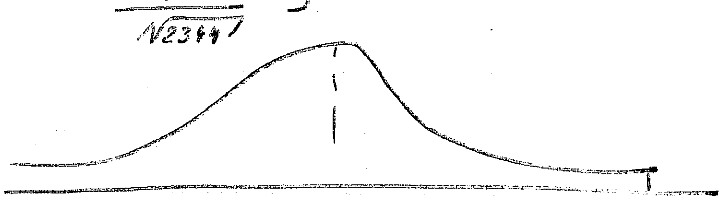
H_0 verwerpen

(X) P : Z

H_0 : $\mu = 3{,}99$

H_A : $\mu \neq 3{,}99$

$$P\left[\frac{3{,}9 - 3{,}77}{\frac{1{,}06}{\sqrt{2344}}}\right] = 5{,}94$$



P-waarde : 0,000... < $\frac{\alpha}{2} = 0{,}025$ → H_0 verwerpen

---

(9)

H_0 : ~~onafh.~~ repr.

H_A : $\overline{\text{n}}$. repr.

		E'
0 - 17	163	204
18 - 64	676	624
65 +	161	172
	1000	

$8{,}24 + 4{,}33 + 0{,}70$

$\chi^2 = 13{,}27 > \chi_2^{2,\,0,01} = 9{,}21$

→ H_0 verwerpen

(10) a) $r = \frac{SS_{xy}}{\sqrt{SS_{xx} \cdot SS_{yy}}}$ $\qquad r = \frac{4500}{\sqrt{4200 \cdot 5600}} \Rightarrow r = 0{,}928$

b) $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 \cdot x$ $\qquad \bar{y} = 110$, $\bar{x} = 85$

$\hookrightarrow \hat{\beta}_1 = \frac{SS_{xy}}{SS_{xx}} = \frac{4500}{4200} = 1{,}07$

$\hookrightarrow \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \cdot \bar{x} = 110 - 1{,}07 \cdot 85 = 19{,}05$

$\Rightarrow \hat{y} = 19{,}05 + 1{,}07 \cdot x$

(gl: $18{,}93 + 1{,}07x$)

c) Breitbandbeispiel

$\alpha = 0{,}05$

$H_0: \hat{\beta}_1 = 0$ $\qquad \cancel{1{,}07 \pm t^{0{,}025}}$

$H_A: \hat{\beta}_1 \neq 0$

$s = \sqrt{\frac{SSE}{n-2}}$ $\qquad t = \frac{\hat{\beta}_1 - \hat{\beta}_{1,0}}{\frac{s}{\sqrt{SS_{xx}}}} = \frac{1{,}07 - 0}{\frac{14{,}01}{\sqrt{4200}}} = \boxed{t = 4{,}95}$

$SSE = SS_{yy} - \hat{\beta}_1 \cdot SS_{xy}$ $\qquad > \left| t_{4}^{0{,}025} \right| = 2{,}776$

$= 5600 - 1{,}07 \cdot 4500$

$s = 14{,}01$ $\qquad \Rightarrow H_0$ verwerfen

$$\boxed{\begin{aligned} SSE &= 778{,}57 \\ s &= 13{,}95 \end{aligned}}$$

$\alpha = 0{,}05$

d) 90% BBI:

$BI_{90\%}$: $1{,}071 \pm t_4^{0{,}05} \cdot \frac{\hat{s}}{\sqrt{SS_{xx}}}$

$= 1{,}071 \pm 2{,}132 \cdot \frac{13{,}95}{\sqrt{4200}} \Rightarrow [0{,}612 \,;\, 1{,}530]$

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e) $VI_{90\%}$ $x = 100\ m^2$

$\hat{y} = 18{,}93 + 1{,}07x = 125{,}93$

$\to \left[125{,}93 \pm t_4^{0{,}05} \cdot 14{,}01 \cdot \sqrt{1 + \frac{1}{n} + \frac{(x_p - \bar{x})^2}{SS_{xx}}} \right]$

2.132 1,08

$= [93{,}67 \,;\, 158{,}19]$

$\boxed{\Leftrightarrow [93{,}21 \,;\, 158{,}93]}$

Afronding . . .

$\left(126{,}07 \pm 2{,}132 \cdot 13{,}95 \cdot \sqrt{1 + \frac{1}{6} + \frac{(100-85)^2}{4200}} \right)$