

5 Exercise layout (CONTD)

Product	Routing	Forecast Production (units / week)	# trips
1	A-B-C-D-E	200	4
2	A-D-E	300	18
3	A-B-C-E	400	8
4	A-C-D-E	650	13

All products are produced in batches of size 50

a) from - to chart

If there is no direct connection between 2 departments, the parts must be routed through other departments to get to their destination.

→ product 1, 3 and 4 have routing ABCDE, for product 2 routing ADE is possible

→ for products 1, 3, 4

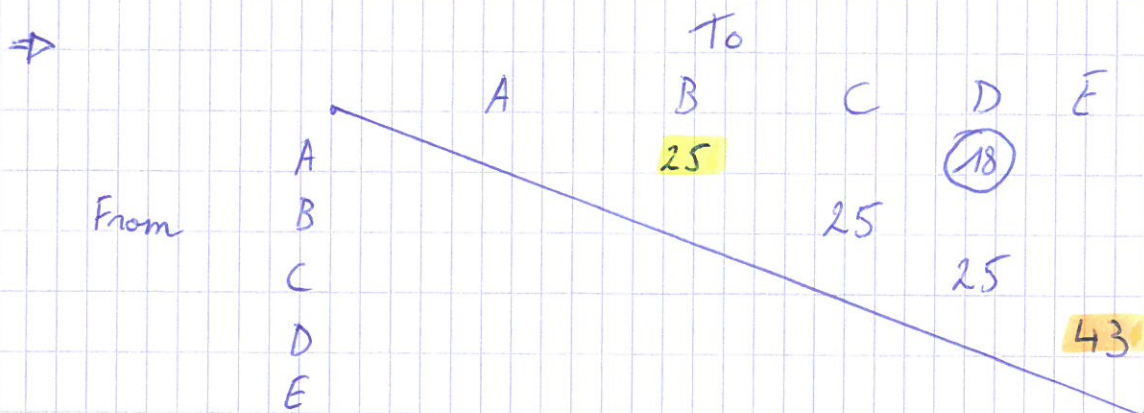
trips of A → B: $4 + 8 + 13 = 25$

A → D: 18

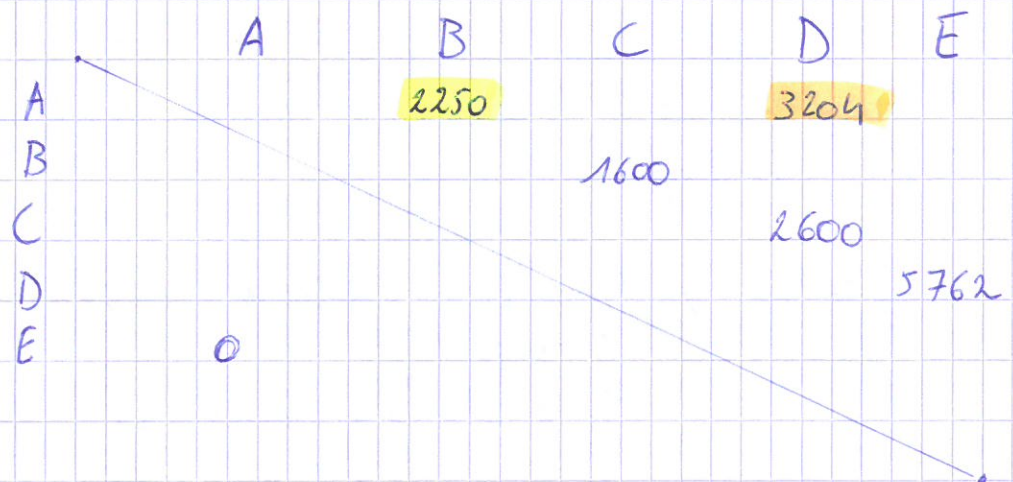
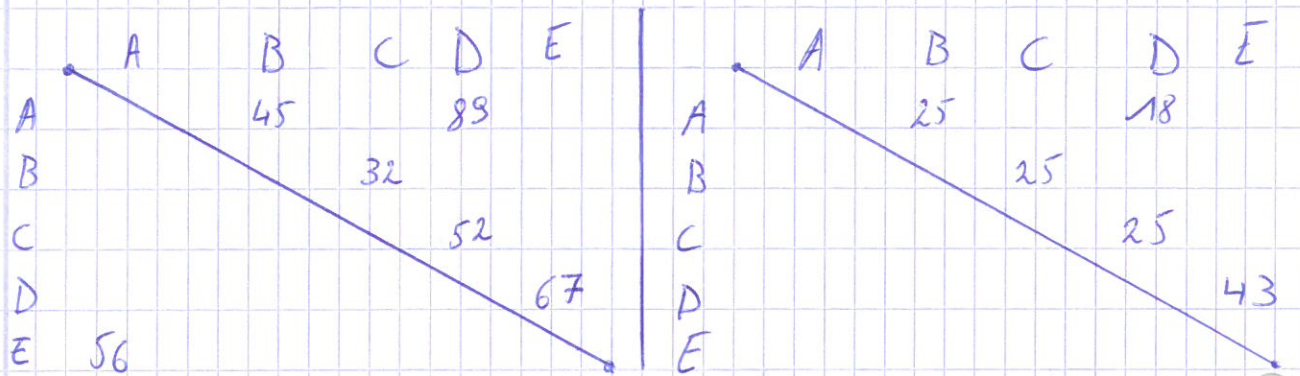
B → C: $4 + 8 + 13 = 25$

C → D: $4 + 8 + 13 = 25$

D → E: $4 + 18 + 8 + 13 = 43$



b) from - to chart giving materials handling cost per week between departments
 + Cost of transporting goods = $\boxed{\$2/\text{foot}}$

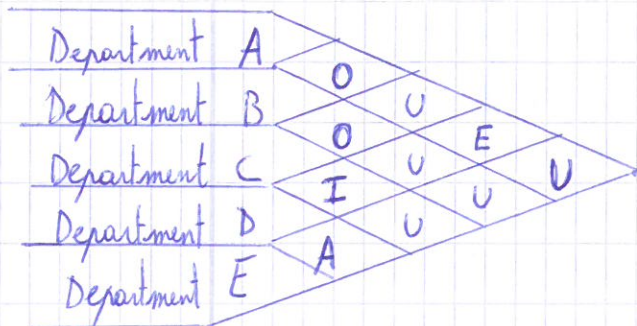


$\text{Yellow box} = 45 \times 25 \times 2 = 2250$

$\text{Orange box} = 89 \times 18 \times 2 = 3204$

c) activity relationship chart. Only use rankings A, E, I, O, U

$\swarrow$ highest cost $\searrow$ lowest cost



	A	B	C	D	E
A		2250		3204	
B			1600		
C				1600	
D					5762
E	0				

A = 5762 $\rightarrow$ highest cost

E = 3204 $\rightarrow$ 2nd

I = 1600 $\rightarrow$ 3rd

O = 2250, 1600 : 2 lowest costs

U $\Rightarrow$ no transport / relation

d) Different layout ?

Yes: move D to the middle: closer to A & E

$\Rightarrow$ lowers the costs of transport between A-D
C-D

Les 5: Location planning and analysis

- 1 Central city location: fixed monthly costs = \$7000
~~Outside~~ " " : transportation costs = \$30/car
Outside location: fixed monthly costs = \$4700
" " : transportation costs = \$40/car

Dealer price = \$90/car

a) 1) demand = 200

$$\begin{aligned} & \bullet \text{ City : } 200 (90 - 30) - 7000 = 5000 \\ & \bullet \text{ Outside : } 200 (90 - 40) - 4700 = 5300 \\ & \rightarrow \text{outside is better} \end{aligned}$$

2) demand = 300

$$\begin{aligned} & \bullet \text{ City : } 300 (90 - 30) - 7000 = 11000 \\ & \bullet \text{ Outside : } 300 (90 - 40) - 4700 = 10300 \\ & \rightarrow \text{City is better} \end{aligned}$$

b) Same monthly profit for which volume of output?

$$Q(90 - 30) - 7000 = Q(90 - 40) - 4700$$

$\rightarrow$ when both functions equal to each other?

$$60Q - 7000 = 50Q - 4700$$

$$10Q = 2300$$

$$Q = 230 \text{ cars}$$

2

a)

Factor	Location score		
	A	B	C
(1) Business services	3	5	5
(2) Community "	7	6	7
(3) Real estate cost	3	8	7
(4) Construction "	5	6	5
(5) Cost of living	4	7	8
(6) Taxes	5	5	4
(7) Transportation	6	7	8

Which one is best when all factors are weighted equally?

Sum of factors : $A = 3 + 7 + 3 + 5 + 4 + 5 + 6 = 33$
 $B = 44$
 $C = 44$

Average of factors : $A = 33/7 = 4,71$
 $B = 44/7 = 6,29$
 $C = 44/7 = 6,29$

$\Rightarrow$ Location B or C is best, followed by A.

b)

Factor	Weight	A	B	C
(1)	2/9	18/9	10/9	10/9
(2)	1/9	7/9	6/9	7/9
(3)	1/9	3/9	8/9	7/9
(4)	2/9	10/9	12/9	10/9
(5)	1/9	4/9	7/9	8/9
(6)	1/9	5/9	5/9	4/9
(7)	1/9	6/9	7/9	8/9
= Sum	1	53/9	55/9	54/9

Weighted average of factors

$\Rightarrow$ Location B is best, followed by C and then A.

③ Center of gravity : which one is the better site ?

Destination	Quantity
D1 (1,2)	900
D2 (2,4)	300
D3 (3,1)	700
D4 (4,2)	600
D5 (5,3)	1200
(x_i, y_i)	<u>3700</u>
(coordinates)	

- Center of gravity

$$\bar{x} = \frac{\sum x_i Q_i}{\sum Q_i} = \frac{1 \cdot 900 + 2 \cdot 300 + 3 \cdot 700 + 4 \cdot 600 + 5 \cdot 1200}{3700} = 3,2$$

$$\bar{y} = \frac{\sum y_i Q_i}{\sum Q_i} = \frac{2 \cdot 900 + 4 \cdot 300 + 1 \cdot 700 + 2 \cdot 600 + 3 \cdot 1200}{3700} = 2,3$$

- Use the map to determine the coordinates for L1 & L2. Use ruler if necessary

L1 coordinates $\approx (2,6 ; 2,4)$

L2 " $\approx (3,5 ; 2,5)$

And $(\bar{x}, \bar{y}) = (3,2 ; 2,3)$

$\Rightarrow$

$$\left\{ \begin{array}{l} \text{for } L_1 : \sqrt{(2,6-3,2)^2 + (2,4-2,3)^2} = 0,608 \\ \text{for } L_2 : \sqrt{(3,5-3,2)^2 + (2,5-2,3)^2} = 0,361 \end{array} \right.$$

How far from center of gravity?
 $= \sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}$

$\Rightarrow$ Which site is closer to the center of gravity (lowest value!)?

$\Rightarrow$ L2 is closer to the center of gravity

Les 6: Inventory - deterministic

[1] ABC-classification

Monthly usages

Category A: (10-15%) of the items with highest value
 B: $\pm 35\%$ of the items
 C: $\pm 50\%$ of the items with lowest value

Item	Usage	Unit Cost	1) Monthly dollar value
4021	50	\$ 1400	\$ 126 000 ✓
8402	300	12	3600 ✓
4066	30	700	21 000 ✓
6500	150	20	3000 ✓
8280	10	1020	10200 ✓
4050	80	140	11200 ✓
6850	2000	10	20 000 ✓
3010	400	20	8000 ✓
4400	5000	5	25 000 ✓

i) Arrange monthly dollar values in descending order

Item	Monthly dollar value	Category	% of items	% of monthly dollar value
4021	\$ 126 000	A	1/9 = 11,11%	$\frac{126000}{228000} = 55,26\%$
4400	25 000	B	3/9 = 33,33%	$\frac{66000}{228000} = 28,95\%$
4066	21 000			
6850	20 000			
4050	11 200	C	5/9 = 55,56%	$\frac{36000}{228000} = 15,79\%$
8280	10 200			
3010	8000			
8402	3600			
6500	3000			
	228 000		100%	100%

9 items $\Rightarrow$ 1 item = $1/9 = 11,11\%$

2) EOQ (Economic Order Quantity)

8000 pots/year

Usage: 750³ pots/month = D

Purchase cost: \$2 / pot = P

Carrying costs per pot = 30% of cost (annual) = $0,3 \times 2 = 0,6$ \$ $\frac{\text{per unit}}{\text{year}}$ $\uparrow$ H

Ordering costs = \$20 per order = S

Order size of 1500 flower pots = Q

a) Additional annual cost?

1) EOQ?

$$*) Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \cdot (8000) \cdot 20}{0,60}} = 775 \text{ pots}$$

$$*) TC = \frac{Q}{2} \cdot H + \frac{D}{Q} \cdot S = \frac{775}{2} \times 0,60 + \frac{8000}{775} \times 20$$

$\downarrow$
with Q_0

$$= \$464,76$$

$$*) TC = \frac{Q}{2} \cdot H + \frac{D}{Q} \cdot S = \frac{1500}{2} \times 0,60 + \frac{8000}{1500} \times 20$$

$\downarrow$
with order size given

$$= \$570$$

$$\Rightarrow \text{additional annual cost: } \$570 - \$464,76 = \$105,24$$

b) Other benefits?

$\Rightarrow$ benefit of using EOQ is that about one half of the storage space would be needed (you can use the other half for something else)

3) EPQ (Economic Production Quantity)

$$u = \text{usage rate} = 20 \text{ tons/day} = 2000 \times 20 = 40\,000 \frac{\text{pounds}}{\text{day}} \quad \downarrow /1000 \\ = 400 \text{ bags per day}$$

$$p = \text{production rate} = 50 \text{ tons/day} = 2000 \times 50 = 100\,000 \frac{\text{pounds}}{\text{day}} \quad \downarrow /1000 \\ = 1000 \text{ bags per day}$$

$$D = 400 \frac{\text{bags}}{\text{day}} \times 200 \frac{\text{days}}{\text{year}} = 80\,000 \text{ bags/year}$$

$$S = 100 \$$$

$$H = \$5/\text{ton per year} \Rightarrow \frac{\$5 \text{ per ton per year}}{20 \text{ bags per ton}} \Rightarrow 2000/100 \\ = \$0,25 \text{ per bag per year}$$

a) How many bags are optimal?

$$Q_p = \sqrt{\frac{2DS}{H}} \times \sqrt{\frac{p}{p-u}} = \sqrt{\frac{2(80\,000) \cdot 100}{0,25}} \times \sqrt{\frac{1000}{1000-400}} \\ = 10327,97 = 10328 \text{ bags}$$

b) Average inventory for this lot size

$$I_{\max} = \frac{Q_p}{p} (p-u) = \frac{10328}{1000} (1000-400) = 6196,8 \text{ bags}$$

$$\Rightarrow I_{\text{average}} = \frac{I_{\max}}{2} = \frac{6196,8}{2} = 3098,4 \text{ bags}$$

c) Run lenght?

$$\text{Run time} = \frac{Q_n}{\uparrow} = \frac{10328}{1000} = 10,33 \text{ days}$$

d) Runs per year $\frac{D}{Q} = \frac{80000}{10328} = 7,75 \text{ runs per year}$

e) Setup cost reduced to \$25 per run? Annual saving?

(1) ($S = \$25$) $TC_{\min} = \left(\frac{I_{\max}}{2}\right) \cdot H + \left(\frac{D}{Q}\right) \cdot S = \left(\frac{3098,4}{2}\right) \times 0,25 + \frac{80000}{5164} = 774,60$

($S = \$25$) $Q_n = \sqrt{\frac{1 \cdot 80000 \cdot 25}{0,25}} \sqrt{\frac{1000}{1000-400}} = 5164 \text{ bags}$

($S = \$25$) $I_{\max} = \frac{5164 (1000-400)}{1000} = 3098,4$

(2) ($S = \$100$) $TC_{\min} = \left(\frac{I_{\max}}{2}\right) \cdot H + \left(\frac{D}{Q}\right) \cdot S$
 $= \left(\frac{6136,8}{2}\right) \times 0,25 + \left(\frac{80000}{10328}\right) \cdot 100 = 774,60 + 774,59$
 $= 1549,19$

$\Rightarrow \text{Annual savings} = 1549,19 - 774,60 = 774,59$

4 Quantity discounts

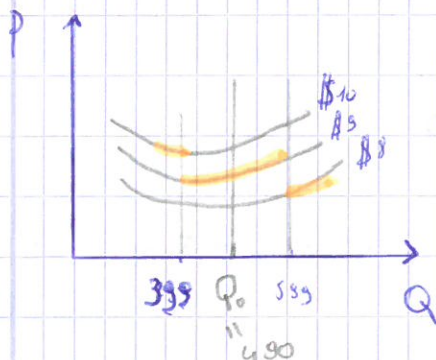
$$u = 25 \frac{\text{stones}}{\text{day}}$$

$$D = 25 \frac{\text{stones}}{\text{day}} \times 200 \frac{\text{days}}{\text{year}} = 5000 \text{ stones/year}$$

$$S = \$48$$

# orders	price (P)
1 - 399	\$10
400 - 599	\$9
600 +	\$8

a) $H = \$2$ per year per stone $\Rightarrow$ Quantity that minimizes total annual cost?



$$\Rightarrow Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \cdot 5000 \cdot 48}{2}} = 490 \text{ stones}$$

$\Rightarrow$ Determine TC for minimum point and for price breaks of all lower unit costs.

$\Rightarrow$ Check total cost for 490 stones and 600 stones (higher # orders, with higher discount) to see which one is lower

$$\begin{aligned} *) TC(490) &= \frac{Q}{2} H + \frac{D}{Q} S + P D = \frac{490}{2} \times 2 + \frac{5000}{490} \times 48 + 9 \times 5000 \\ &= 45979,80 \end{aligned}$$

$$\begin{aligned} *) TC(600) &= \frac{Q}{2} H + \frac{D}{Q} S + P D = \frac{600}{2} \times 2 + \frac{5000}{600} \times 48 + 8 \times 5000 \\ &= 41000 \end{aligned}$$

$\Rightarrow$ Conclusion: Optimal order quantity = 600 stones

b) carrying costs = 30% of unit cost $\Rightarrow$ Optimal order size?

$$H = 0,3 \cdot p$$

(1) Beginning with the lowest unit price, compute the minimum points for each price range, until you find a feasible minimum point

* Minimum point for $p = \$8$

$$Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \cdot 5000 \cdot 48}{0,3 \cdot 8}} = 447,21 = 447$$

Not feasible, since this quantity is not higher than 600!
 $\Rightarrow$ need at least a size of 600 to get the price of $\frac{\$8}{\text{unit}}$

* Minimum point for $p = \$9$

$$Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \cdot 5000 \cdot 48}{0,3 \cdot 9}} = 421,64 = 422 = \text{feasible}$$

since this is in the order range of $p = \$9$ (400 - 599)

(2) Compare the total cost at $Q = 422$ to $Q = 600$

$$TC_{422} = \frac{Q}{2} H + \frac{D}{Q} S + PD = \frac{422}{2} \times (0,3 \times 9) + \left(\frac{5000}{422}\right) \times 48 + 9 \times 5000 = 46138,42$$

$$TC_{600} = \frac{Q}{2} H + \frac{D}{Q} S + PD = \frac{600}{2} \times (0,3 \times 8) + \frac{5000}{600} \times 48 + 8 \times 5000 = 41120$$

$\Rightarrow$ Conclusion: optimal order quantity = 600 stones

c) ROP (reorder point)

$$ROP = d \times LT$$

d = demand rate

LT = lead time in days/weeks

$$\begin{aligned} ROP &= 25 \text{ stones/day} \times 6 \text{ days} \\ &= 150 \text{ stones.} \end{aligned}$$

des 7: Inventory: stochastic

- 1 Usage rate = 800 feet/day
LT = 6 days
Service level = 95%

Continuous review

ROP?

Safety stock	Stockout risk	Service level
1500	0,10	0,90
1800	0,05	0,95
2100	0,02	0,98
2400	0,01	0,99

→ To have a service level of 95%, a safety stock of 1800 is needed.

$$\begin{aligned}\rightarrow \text{ROP} &= \text{expected demand during lead time} + \text{safety stock} \\ &= 6 \times 800 + 1800 = 6600 \text{ feet.}\end{aligned}$$

2 Service level

Expected demand during LT = 300 units
 σ_{dLT} = Standard deviation of LT demand = 30 units
LT = distributed normally

a) ROP with 1% risk of stockout during LT $\Rightarrow$ 1% risk $\Rightarrow$ 99% service level

$$\begin{aligned}\text{ROP} &= \text{expected demand during LT} + \text{safety stock} \\ &= 300 + 2 \cdot \sigma_{\text{dLT}} \\ &= 300 + 2 \cdot 30 \\ &= 300 + 2,33 \cdot 30 \\ &= 369,9 \approx 370 \text{ (round up)}\end{aligned}$$

$\Downarrow$
 $z = 2,33$
(closest to 99%
 $= 0,9901$)
 $\Downarrow$
find this in
the table

b) safety stock needed to have a 1% risk of stockout during LT

Service level = 99%

$$\begin{aligned} \hookrightarrow \text{Safety stock} &= z \cdot \sigma_d \cdot \sqrt{LT} \\ &= 2,33 \times 30 = 69,9 \approx 70 \end{aligned}$$

c) Service level of 98% is lower, so ^{less} safety stock would be needed and the ROP would be smaller.

3 Continuous and periodic review

- Normal distribution (Demand) (week in weeks)
- Mean = 21 gallons/week (Demand) $\rightarrow$ 3 gallons/day
- $\sigma = 3,5$ gallons/week (Demand)
- Service level needed = 90%
- $LT = 2 \text{ days} = \frac{2}{7} \text{ week}$
- Open: 7 days a week

Closest to 90%
 $\uparrow$

a) ROP? Desired service level = 90% $\rightarrow z = 1,28$ ($F(1,28) = 0,8997$)

Demand = variable, $LT = \text{constant}$

$$\begin{aligned} \Rightarrow ROP &= \bar{d} \times LT + z \cdot \sigma_d \cdot \sqrt{LT} \\ &= \text{exp. dem. during } LT + \text{safety stock} \end{aligned}$$

$$= 21 \times \frac{2}{7} + 1,28 \cdot 3,5 \sqrt{\frac{2}{7}} = 8,39 \text{ gallons} \approx 9$$

$\Rightarrow$ How many days of supply are on hand at ROP?

$$= \frac{ROP}{\text{Average daily demand}} = \frac{9}{3} = 3 \text{ days}$$

b-) Fixed-interval model • Size needed for 90% service level with order interval of 10 days and supply of 8 gallons on hand at the time of order?

→ Q?

→ probability of stockout before this order arrives?

Q = expected demand during protection interval + safety stock - amount on hand at reorder point (A)

$$*) Q = \bar{d} (OI + LT) + z \sigma_d \sqrt{OI + LT} - A$$

OI = order interval

$$Q = 21 \times \left(\overset{\text{in weeks}}{\frac{10+2}{7}} \right) + 1,28 \times 3,5 \times \sqrt{\frac{10+2}{7}} - 8$$

$$= 33,87 \text{ gallons} \approx \boxed{34}$$

*) ROP (demand uncertain)

$$R = \bar{d} \times LT + z \sigma_d \sqrt{LT}$$

$$8 = 21 \times \frac{2}{7} + z \times 3,5 \times \sqrt{2/7} \Rightarrow z = \frac{2}{1,871} = 1,07$$

They reorder when they have 8 gallons left

See table

Lead time service level = 85,77%

c) ROP model:

After one day, order will be 2 days later + 2 gallons have been sold since the order had been placed

After 1 day, quantity on hand = $9 - 2 = 7$ gallons
 ↳ probability of running out of stock?

$$ROP = 7 = \bar{d} \times LT + z \sigma_d \sqrt{LT}$$

$$= 21 \times \frac{2}{7} + z \times 3,5 \times \sqrt{2/7}$$

$$\Rightarrow z = \frac{1}{1,871} = 0,53 \xrightarrow{(*)} \text{lead time service level is } 0,7019$$

(*) see figure

$$\Rightarrow \text{risk of stockout} = 1 - 0,7019 = 29,81\%$$

[4] Newsboy problem with discrete distribution

Demand	0	1	2	3
Probability	0,15	0,35	0,30	0,20

a) Ratio method

$$C_s = \text{revenue/unit} - \text{cost/unit}$$

$$= 60 - 33$$

$$= 27$$

$$C_e = \text{original cost/unit} - \text{salvage value/unit}$$

$$= 33 - \left(\frac{1}{2} \times \frac{1}{3} \times 60\right)$$

$$= 23$$

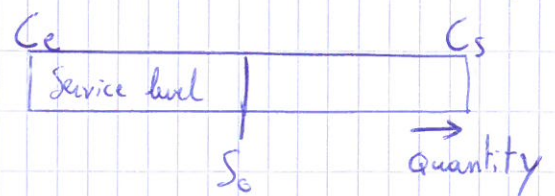
Demand	0	1	2	3
Cumul. Prob.	0,15	0,50	0,8	1,00

$$SL = \frac{C_s}{C_s + C_e} = \frac{27}{27 + 23} = 0,54 = \text{between } 1 \text{ \& } 2$$

$\Rightarrow$ Stock 2 cakes

$$\text{Service level} = \frac{C_s}{C_s + C_e}$$

where C_s = shortage cost/unit
 C_e = excess cost/unit



Balance point
 = Optimum stocking quantity

(37)

b) Tabular method

I / the stocking level is	Demand = 0 prob. = 0,15	Demand = 1 prob. = 0,35	Demand = 2 prob. = 0,30	Demand = 3 prob. = 0,20	Expected payoff
0	Sell 0, over 0 $S = D$ \$0	Sell 0, over 0 $(0,35 \times 0 \times 27)$ $-(0,35 \times 0 \times 23)$ = \$0	Sell 0, over 0 $(0,3 \times 0 \times 27)$ $-(0,3 \times 0 \times 23)$ = \$0	Sell 0, over 0 $(0,20 \times 0 \times 27)$ $-(0,2 \times 0 \times 23)$ = \$0	\$0
1	Sell 0, over 1 $(0,15 \times 0 \times 27)$ $-(0,15 \times 1 \times 23)$ = -\$3,45	Sell 1, over 0 $(0,35 \times 1 \times 27)$ $-(0,35 \times 0 \times 23)$ = \$9,45	Sell 1, over 0 $(0,30 \times 1 \times 27)$ $-(0,30 \times 0 \times 23)$ = \$8,10	Sell 1, over 0 $(0,20 \times 1 \times 27)$ $-(0,20 \times 0 \times 23)$ = \$5,40	\$19,50 = sum of
2	Sell 0, over 2 $(0,15 \times 0 \times 27)$ $-(0,15 \times 1 \times 23)$ = -\$6,90	Sell 1, over 1 $(0,35 \times 1 \times 27)$ $-(0,35 \times 1 \times 23)$ = \$1,40	Sell 2, over 0 $(0,30 \times 2 \times 27)$ $-(0,30 \times 0 \times 23)$ = \$16,20	Sell 2, over 1 $(0,20 \times 2 \times 27)$ $-(0,20 \times 1 \times 23)$ = \$10,80	\$21,50
3	Sell 0, over 3 $(0,15 \times 0 \times 27)$ $-(0,15 \times 3 \times 23)$ = -\$10,35	Sell 1, over 2 $(0,35 \times 1 \times 27)$ $-(0,35 \times 2 \times 23)$ = -\$6,65	Sell 2, over 1 $(0,30 \times 2 \times 27)$ $-(0,30 \times 1 \times 23)$ = \$9,30	Sell 3, over 0 $(0,20 \times 3 \times 27)$ $-(0,20 \times 0 \times 23)$ = \$16,20	\$8,50

⇒ Stock 2 cakes!

5) Newsboy problem with normal distribution

- * 18 ticketholders cancel (on average)
 - * Cancellations $\approx$ normal distribution with a mean of 18 and standard deviation of 4,55
 - * Profit / passenger = \$99
 - * Cash payment of \$200 if it's ^{actually} overlooked (for 1 seat)
- How many tickets should be overlooked?
Note: a cancellation = demand for overlooking

$C_s = \$99$ = cost you lose if you do not have enough tickets overlooked
 $C_e = \$200$ = cost you have if there are too many tickets overlooked.

$$\text{Ratio: } SL = \frac{C_s}{C_s + C_e} = \frac{99}{99 + 200} = 0,3311 \rightarrow z = -0,44$$

$$\begin{aligned} S_0 &= \underline{\text{mean}} + z \cdot \underline{\sigma} \\ &= 18 + (-0,44 \times 4,55) = 15,998 \approx 16 \text{ tickets to overlook} \end{aligned}$$

6) Newsboy problem

4 machines
profit = \$ 10/day

Demand	Frequency
0	0,3
1	0,2
2	0,2
3	0,15
4	0,10
5	0,05
	<u>1,00</u>

a)	Demand	0	1	2	3	<u>4 machines</u>	4	5
	Cumul. prob.	0,30	0,5	0,7	0,85	0,95	1,00	

⇒ for 4 machines to be optimal, the SL ratio must be between $> 0,85$ and $\leq 0,95$

$$SL = \frac{10}{10 + C_e} = 0,85 \rightarrow C_e = \$1,76$$

$$SL = \frac{10}{10 + C_e} = 0,95 \rightarrow C_e = \$0,53$$

⇒ Implied range of excess cost: $0,53 < C_e < 1,76$

b) C_e range is judged to be too low. If C_e is actually higher, having too much is more costly, and the number of machines should be decreased. The service level will also decrease - optimal