EXTRA OEFENINGEN

Wiskunde I voor bedrijfskundigen

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Extra opgaven

OPGAVE 1

a. $f(x) = \dfrac{x-3}{\sqrt{x^2-3x+2}}$

voorwaarden: $\begin{cases} \sqrt{x^2-3x+2} \neq 0 \\ x^2-3x+2 \geq 0 \ * \end{cases}$

$* \ D = b^2 - 4.a.c \rightsquigarrow (-3)^2 - 4.1.2 = 1$

$\dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{3 \pm \sqrt{1}}{2} \begin{matrix} \nearrow 1 \\ \searrow 2 \end{matrix}$

		1		2	
x^2-3x+2	+	0	−	0	+

beperken door voorwaarde 1

$\text{dom } f = \left]-\infty ; 1\right[\cup \left]2 ; +\infty\right[$

b. $\dfrac{\sqrt{x-2}}{\sqrt[4]{x^2-2x}}$

voorwaarden: $\begin{cases} x-2 \geq 0 \ * \\ x^2-2x \geq 0 \\ \sqrt[4]{x^2-2x} \neq 0 \ ** \end{cases}$

$* \ x-2 \geq 0 \Leftrightarrow x-2 = 0 \Leftrightarrow x = 2$

		2	
$x-2$	−	0	+

$** \ x^2-2x \geq 0 \Leftrightarrow x^2-2x = 0$

$D = (-2)^2 - 4.1.0 = 4$

$\dfrac{2 \pm 2}{2} \begin{matrix} \nearrow 0 \\ \searrow 2 \end{matrix}$

		0		2	
x^2-2x	+	0	−	0	+

dom f = $]2;+\infty[$ rekening houdend met beperking *

c. $\dfrac{x}{\sqrt{x^2+4}}$

voorwaarden $\begin{cases} \sqrt{x^2+4} \neq 0 \\ x^2+4 \geq 0 \; * \end{cases}$

$$* \; x^2+4=0$$
$$x^2=-4$$
$$x=\pm\cancel{\sqrt{-4}}$$ kan niet!!

dom f = $\mathbb{R}$

d. $f(x)=\dfrac{x+4}{x.(x^2-16)}$

voorwaarden? $x.(x^2-16)\neq 0$

$$\Downarrow \qquad \Downarrow$$
$$x\neq 0 \qquad x\neq 4 \text{ en } -4$$

dom f = $\mathbb{R}/\{-4,0,4\}$

e. $f(x)=\log_3\left(\dfrac{\sqrt{x.(x-3)}}{x}\right)$

voorwaarden $\begin{cases} \left(\dfrac{\sqrt{x.(x-3)}}{x}\right) > 0 \; ^{(1)} \\ x.(x-3)\geq 0 \\ x\neq 0 \end{cases}$

*** $x \neq 0$

** $x.(x-3) \geq 0$

$\Downarrow \quad \Downarrow$

$x \neq 0 \quad x \neq 3$

		0		3	
x	$-$	0	+	+	+
$x-3$	$-$	$-$	$-$	0	+
	+	0	$-$	0	+

beperking door log!! (1)

dom $f =]3;+\infty[$

f. $\sqrt{4-x} + \ln(-x^2+8x-12)$

voorwaarden: $4-x \geq 0$ *

$-x^2+8x-12 > 0$ **

* $4-x = 0 \Leftrightarrow x = 4$

		4	
$4-x$	+	0	$-$

** $8^2 - 4.(-1).12 = 16$

$\frac{-8 \pm \sqrt{16}}{-2}$ ↗ 6 ↘ 2

		2		6	
$-x^2+8x-12$	$-$	0	+	0	$-$

rekening houden beperking *

dom $f =]2;4]$

g. $f(x) = \dfrac{\log(x+2)}{x^2+3x-4}$

voorwaarden $\begin{cases} x+2>0 \quad {}^{*} \\ x^2+3x-4 \neq 0 \end{cases}$

* $x+2=0 \leftrightarrow x=-2$

		-2	
$x+2$	$-$	0	$+$

** $3^2 - 4 \cdot 1 \cdot (-4) = 25$

$\dfrac{-3 \pm \sqrt{25}}{2}$ $\nearrow -4$ $\searrow 1$

		-4		1	
x^2+3x-4	$+$	0	$-$	0	$+$

rekening houden beperking *

dom $f =]-2; +\infty[\, / \{1\}$

h. $f(x) = \sqrt{\dfrac{4x^2-9}{1-9x^2}}$

voorwaarden $\begin{cases} \left(\dfrac{4x^2-9}{1-9x^2}\right) \geq 0 \quad {}^{*} \\ 1-9x^2 \neq 0 \quad {}^{**} \end{cases}$

** $1-9x^2=0$

$9x^2 = 1$

$x^2 = 1/9$

$x = \pm 1/3$

		$-\frac{1}{3}$		$\frac{1}{3}$	
$-9x^2+1$	$-$	0	$+$	0	$-$

$^{\times}\ 4x^2 - 9 = 0$

$4x^2 = 9$

$x^2 = 9/4$

$x = \pm 3/2$

([illegible])

		$-\frac{3}{2}$		$-\frac{1}{3}$		0		$\frac{1}{3}$		$\frac{3}{2}$	
$4x^2-9$	+	0	−	−	−	−	−	−	−	0	+
$1-9x^2$	−	−	−	0	+	+	+	0	−	−	−
	−	0	+	0	−	−	−	0	+	0	−

$\text{dom } f = [-3/2 ; -1/3[\cup]1/3 ; 3/2]$

OPGAVE 2 $\quad f(x) = \dfrac{1}{\sqrt{2-3x}} + 5$

a. voorwaarden: $\begin{cases} \sqrt{2-3x} \neq 0 \\ 2-3x \geq 0 \ ^{\times} \end{cases}$

$^{\times}\ 2 - 3x = 0$

$-3x = -2$

$x = 2/3$

		$2/3$	
$2-3x$	+	0	−

$\text{dom } f =]-\infty ; 2/3]$

b. $x = \dfrac{1}{\sqrt{2-3y}} + 5$

$x - 5 = \dfrac{1}{\sqrt{2-3y}}$

!! $\sqrt{2-3y}^{\,2} = \left(\dfrac{1}{(x-5)}\right)^2 \Leftrightarrow 2 - 3y = \dfrac{1}{(x-5)^2} \quad \Leftrightarrow -3y = \dfrac{1}{(x-5)^2} - 2$

$\Leftrightarrow y = \dfrac{-1}{3.(x-5)^2} + \dfrac{2}{3}$ beperking? $(3.(x-5)^2) \neq 0$ dus $x \neq 5$

c. $\frac{1}{\sqrt{2-3x_1}} + 5 = \frac{1}{\sqrt{2-3x_2}} + 5$

$\Leftrightarrow \sqrt{2-3x_1} + \frac{1}{5} = \sqrt{2-3x_2} + \frac{1}{5}$

$\Leftrightarrow 2-3x_1 + \frac{1}{25} = 2-3x_2 + \frac{1}{25}$

$\Leftrightarrow 2-3x_1 + \cancel{\frac{1}{25}} - \cancel{\frac{1}{25}} = 2-3x_2$

$\Leftrightarrow \cancel{2} - 3x_1 - \cancel{2} = -3x_2$

$\Leftrightarrow -\cancel{3}x_1 = -\cancel{3}x_2$

$\rightsquigarrow$ INJECTIE

d. $y = \frac{-1}{3.(x-5)^2} + \frac{2}{3}$

OPGAVE 5

a. $W = O - K \rightsquigarrow p.q - K$

$-\frac{2}{5}q^2 + 10q - \frac{1}{10}q^2 - 8 = \boxed{-\frac{1}{2}q^2 + 10q - 8}$

b. top $\left(\frac{+10}{\cancel{2}.\left(+\frac{1}{\cancel{2}}\right)}, 42\right)$

c. $10^2 - 4.\left(-\frac{1}{2}\right).8 = 84$

$\frac{+10 \pm \sqrt{84}}{\cancel{2}.\left(+\frac{1}{\cancel{2}}\right)} = x$

d. $-\frac{2}{5}q^2 + 10q$

$10^2 - 4\left(-\frac{2}{5}\right).0 = 100$

$\frac{-10 \pm \sqrt{100}}{2.\left(-\frac{2}{5}\right)}$ $\nearrow 0$ $\searrow 25$ $\rightsquigarrow$ dom f $[0,25]$

OPGAVE 3

a. $f(x) = x + \log_5(5^x - 4.5^{-x}) - 1$

$\log_5(5^x - 4.5^{-x}) = 1 - x$

als onbekende exponent en log [illegible]!

$\Leftrightarrow \log_5(5^x - 4.5^{-x}) = \log_5 5^{1-x}$

$\Leftrightarrow 5^x - 4.5^{-x} = 5^{1-x}$

$\Leftrightarrow 5^x - 4.5^{-x} = 5.5^{-x}$

$\Leftrightarrow \frac{5^x}{5^{-x}} - \frac{4.\cancel{5^{-x}}}{\cancel{5^{-x}}} = \frac{5.\cancel{5^{-x}}}{\cancel{5^{-x}}}$

$\Leftrightarrow 5^{2x} - 4 = 5$

$\Leftrightarrow 5^{2x} = 9$

$\Leftrightarrow \log 5^{2x} = \log 9$

$\Leftrightarrow 2x \log 5 = \log 9$

$\Leftrightarrow 2x = \frac{\log 9}{\log 5}$

$\Leftrightarrow 2x = \frac{\log 3^2}{\log 5}$

$\Leftrightarrow \cancel{2}x = \cancel{2}.\frac{\log 3}{\log 5}$

b. $f(x) = 2^{2x+2} - \frac{7}{2}.2^{x+1} - 2 \quad \downarrow .2$

$\Leftrightarrow 2.2^{2x+2} - 7.2^{x+1} - 4 = 0$

$\Leftrightarrow 2.2^{2x}.2^2 - 7.2.2^x - 4 = 0$

$\Leftrightarrow 2^3.2^{2x} - 14.2^x - 4 = 0$

$\Leftrightarrow 8.(2^x)^2 - 14.2^x - 4 = 0$

stel $2^x = t$

$\Leftrightarrow 8t^2 - 14t - 4 = 0$

$D = (-14)^2 - 4.8.(-4) = 196 + 128 = 324$

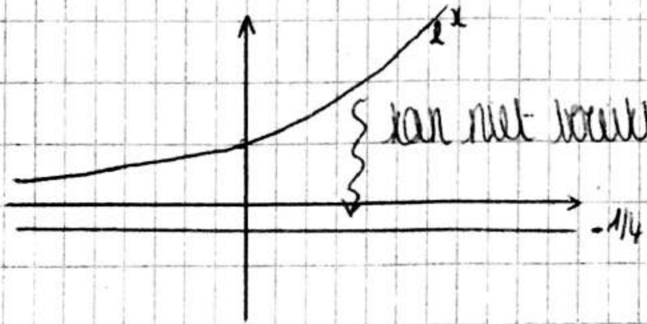
$\frac{14 \pm 18}{16} \nearrow 2 \quad \searrow -1/4$

uitkomst in oorspronkelijke voorwaarde stoppen!

stel $2^x = 2$, dan $x = 1$

stel $2^x = -1/4$ niet mogelijk! kan niet ⊖ zijn!

↳ 2 y-waarden, grafiek tekenen!



c $f(x) = 3(\log_3(1-x))^2 - 5 \cdot \log_3(1-x) - 2$

stel $\log_3(1-x) = t$

$\Leftrightarrow 3t^2 - 5t - 2 = 0$

$\Leftrightarrow D = (-5)^2 - 4 \cdot 3 \cdot -2 = 25 + 24 = 49$

$\frac{5 \pm 7}{6} \to 2$, $\to -1/3$

stel $\log_3(1-x) = 2$

$\Leftrightarrow \log_3(1-x) = \log_3 3^2$

$\Leftrightarrow 1 - x = 9$

$\Leftrightarrow -x = 8$

$x = -8$

stel $\log_3(1-x) = -1/3$

$\Leftrightarrow \log_3(1-x) = \log_3 3^{-1/3}$

$\Leftrightarrow 1 - x = 3^{-1/3}$

$\Leftrightarrow 1 - x = \frac{1}{3^{1/3}}$

$\Leftrightarrow -x = \frac{1}{3^{1/3}} - 1$

$\Leftrightarrow x = \frac{-1}{3^{1/3}} + 1$

OPGAVE 4

a. $\log_5(3x-5) - \log_5(x^2+4x-11) = 0$

$\Leftrightarrow \log_5(3x-5) = \log_5(x^2+4x-11)$

$\Leftrightarrow 3x-5 = x^2+4x-11$

$\Leftrightarrow x^2+x-6=0$

$D = (1)^2 - 4 \cdot 1 \cdot -6 = 25$

$\frac{-1 \pm 5}{2}$ $\nearrow 2$ $\searrow \cancel{-3}$ kan niet $\ominus$ zijn!

b. $3^{\sqrt{x}+1} = 3^{x-5}$

$\Leftrightarrow \sqrt{x}+1 = x-5$

$\Leftrightarrow \sqrt{x} - x = -6$

stel $x = t^2$

$\Leftrightarrow \sqrt{t^2} - t^2 + 6 = 0$

$\Leftrightarrow -t^2 + t + 6 = 0$

$D = 1^2 - 4 \cdot -1 \cdot 6 = 25$

$\frac{-1 \pm 5}{-2}$ $\nearrow -2$ $\searrow 3$

stel $-2 = \cancel{x}^2$ kan niet $\ominus$ zijn

stel $x = 3^2 \rightsquigarrow x = 9$

c. $2 \log x + 1 = \log(19x+2)$

$\Leftrightarrow \log x^2 + \log 10^1 = \log(19x+2)$

$\Leftrightarrow \log(x^2 \cdot 10^1) = \log(19x+2)$

$\Leftrightarrow x^2 \cdot 10^1 = 19x+2$

$\Leftrightarrow 10x^2 - 19x - 2 = 0$

$D = (-19)^2 - 4 \cdot 10 \cdot -2 = 361 + 80 = 441$

$\frac{19 \pm 21}{20}$ $\nearrow 2$ $\searrow -0{,}1$ kan niet $\ominus$ zijn

d. $\ln(2x+3)+\ln(x-1)=\ln(x^2+9)$

$\Leftrightarrow \ln((2x+3)(x-1))=\ln(x^2+9)$

$\Leftrightarrow (2x+3)(x-1)=x^2+9$

$\Leftrightarrow 2x^2-2x+3x-3=x^2+9$

$\Leftrightarrow x^2+x-12=0$

$D=1^2-4.1.-12=49$

$\frac{-1\pm 7}{2}$ $\nearrow 3$ $\searrow -4$ kan niet $\ominus$ zijn!

e. $(\log x)^2-\log x^2-\log_{10}10^{15}=0$

$\Leftrightarrow \log^2 x-2.\log x-15=0$

$\Leftrightarrow$ stel $\log x=t$

$t^2-2t-15=0$

$D=(-2)^2-4\,1\,(-15)=4+60=64$

$\frac{2\pm 8}{2}$ $\nearrow 5$ $\searrow -3$

stel $\log x=5$

$\log x=\log 10^5$

$x=10^5$

stel $\log x=-3$

$\log x=\log 10^{-3}$

$x=10^{-3}$

f. $2^{2x}-2^{x+1}=3$

$2^{2x}-2^x.2=3$

$(2^x)^2-2^x\,2-3=0$

stel $2^x=t$

$t^2-2t-3=0$

$D=(-2)^2-4.1.(-3)=4+12=16$

$\frac{2\pm 4}{2}$ $\nearrow 3$ $\searrow -1$

stel $2^x = 3$

$\log_2 2^x = \log_2 3$

$x = \log_2 3$

stel $2^x = -1$ kan niet $\ominus$ zijn

OPGAVE 6 (oorspronkelijk fout in opl!)

$p_V = -\frac{2}{5}q^2 + 9$

$p_A = \frac{2}{5}q$

nieuwe aanbodsfunctie: $\boxed{\frac{2}{5}q + 1}$

$$-\frac{2}{5}q^2 + 9 = \frac{2}{5}q + 1$$
$$\Leftrightarrow -\frac{2}{5}q^2 - \frac{2}{5}q + 8 = 0$$
$$\Leftrightarrow -2q^2 - 2q + 40 = 0$$
$$D = (-2)^2 - 4.(-2).40$$
$$= 4 + 320 = 324$$

$$\frac{2 \pm 18}{-2.2} \begin{array}{l} \nearrow 4 \\ \searrow \cancel{-5} \end{array}$$ economisch niet relevant

prijs ? $\frac{2}{5}.4 + 1 = 8/5 + 1 = 13/5$

marktomzet ? $= \frac{13}{5}.4 = 52/5$

OPGAVE 7

$$\begin{cases} f(x) = \dfrac{x^3 - 27}{x^2 - 9} & \text{als } x \neq 3 \\ ? & \text{als } x = 3 \end{cases}$$

gegeven: 3 is continu $\rightsquigarrow$ limiet nagaan!

ratio. functie NP ook noemer $\Downarrow$ ontbinden factoren

$$\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9} = \frac{3^3 - 27}{3^2 - 9} = \frac{0}{0}$$

$$\Leftrightarrow \lim_{x \to 3} \frac{\cancel{(x-3)}.(x^2 + 3x + 3^2)}{\cancel{(x-3)}.(x+3)} = \lim_{x \to 3} \frac{x^2 + 3x + 3^2}{x+3} = \frac{3^2 + 3\,3 + 3^2}{3+3} = \frac{\cancel{27}\,9}{\cancel{6}\,2}$$

$A^2 - B^2 = (A+B).(A-B)$

$A^3 - 3^3 = (A-B).(A^2 + AB + B^2)$

$$\lim_{x \overset{<}{\to} 3} \frac{x^3-27}{x^2-9} \Rightarrow \text{idem} \lim_{x \overset{>}{\to} 3}$$

conclusie: "de" limiet bestaat en $f(3) = 9/2$

OPGAVE 8 a $\lim_{x \to 2} \frac{(2x-4) \cdot \sqrt{x+2}}{x^2-4x+4}$ $\left(= \frac{0}{0}\right)$

$$\Leftrightarrow \lim_{x \to 2} \frac{(2x-4) \cdot \sqrt{x+2}}{(x-2) \cdot (x-2)} \Leftrightarrow \lim_{x \to 2} \frac{2 \cdot \cancel{(x-2)} \cdot \sqrt{x+2}}{\cancel{(x-2)} (x-2)}$$

$$\Leftrightarrow \lim_{x \to 2} \frac{2 \cdot \sqrt{x+2}}{(x-2)} = \frac{4}{0}$$

		2	
$x-2$	$-$	0	$+$

linkerlimiet $-\infty$
rechterlimiet $+\infty$

b. $\lim_{x \to +\infty} 2x - \sqrt{4x^2+1}$ (onbepaald: $+\infty - (+\infty)$)

$$\Leftrightarrow \lim_{x \to +\infty} 2x - \sqrt{4x^2+1} \cdot \frac{2x + \sqrt{4x^2+1}}{2x + \sqrt{4x^2+1}}$$

$$\Leftrightarrow \lim_{x \to +\infty} \frac{\cancel{4x^2} - \cancel{4x^2}+1}{2x+\sqrt{4x^2+1}} \Leftrightarrow \lim_{x \to +\infty} \frac{1}{2 \cdot (+\infty) + \sqrt{4 \cdot (+\infty)^2+1}} = \frac{1}{+\infty} = 0$$

c. $\lim_{x \to -\infty} 3x + \sqrt{4x^2+x+1}$ (onbepaald: $\sqrt{+\infty} + (-\infty)$)

$$\Leftrightarrow \lim_{x \to -\infty} 3x + \sqrt{4x^2+x+1} \cdot \frac{3x - \sqrt{4x^2+x+1}}{3x - \sqrt{4x^2+x+1}}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{9x^2-4x^2-x-1}{3x - \sqrt{4x^2+x+1}} = \lim_{x \to -\infty} \frac{5x^2-x-1}{3x - \sqrt{4x^2+x+1}}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{5x^2-x-1}{3x - \sqrt{4x^2 \cdot \left(1 + \frac{x}{4x^2} + \frac{1}{4x^2}\right)}}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{5x^2 - x - 1}{3x - |2x| \cdot \sqrt{1 + \frac{1}{4x} + \frac{1}{4x^2}}} = \lim_{x \to -\infty} \frac{5x^2 - x - 1}{3x + 2x\sqrt{1 + \frac{1}{4x} + \frac{1}{4x^2}}}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{5x^2 - x - 1}{x.\left(3 + 2\sqrt{1 + \frac{1}{4x} + \frac{1}{4x^2}}\right)} = \lim_{x \to -\infty} \frac{x.\left(5x - 1 - \frac{1}{x}\right)}{\cancel{x}.\left(3 + 2\sqrt{1 + \frac{1}{4x} + \frac{1}{4x^2}}\right)}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{5.(-\infty) - 1 - \cancel{\frac{1}{-\infty}}}{3 + 2\sqrt{1 + \cancel{\frac{1}{4 \cdot -\infty}} + \cancel{\frac{1}{4(-\infty)^2}}}} = \lim_{x \to -\infty} \frac{-\infty - 1}{5} = -\infty$$

d. $\lim\limits_{x \to -\infty} \sqrt{x^2 - 2x} + x$ (onbepaald: $\sqrt{+\infty} + (-\infty)$)

$$\Leftrightarrow \lim_{x \to -\infty} \sqrt{x^2 - 2x} + x \cdot \frac{\sqrt{x^2 - 2x} - x}{\sqrt{x^2 - 2x} - x} = \lim_{x \to -\infty} \frac{\cancel{x^2} - 2x - \cancel{x^2}}{\sqrt{x^2 - 2x} - x}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{-2x}{\sqrt{x^2 \cdot \left(1 - \frac{2x}{x^2}\right)} - x} = \lim_{x \to -\infty} \frac{-2x}{-x.\sqrt{1 - \frac{2}{x}} - x}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{-2\cancel{x}}{\cancel{x}.\left(-\sqrt{1 - \frac{2}{x}} - 1\right)} = \lim_{x \to -\infty} \frac{-2}{-\sqrt{1 - \cancel{\frac{2}{-\infty}}} - 1} = \frac{-2}{-\sqrt{1} - 1} = \frac{-2}{-2} = 1$$

e. $\lim\limits_{x \to 1} 0{,}22^{\frac{1}{(1-x)^3}}$

$$\Leftrightarrow \lim_{x \to 1} e^{\frac{1}{(1-x)^3} \ln 0{,}22} = e^{\lim\limits_{x \to 1} \ln 0{,}22 \cdot \frac{1}{(1-x)^3}}$$

$$\Leftrightarrow e^{\ln 0{,}22 \lim\limits_{x \to 1} \frac{1}{(1-x)^3}}$$

$$\Updownarrow$$

$$\lim_{x \to 1} \frac{1}{(1-x)^3} = \frac{1}{0}$$

	0		1	
$(1-x)^3$	+		0	$\cancel{+}$ −

linkerlimiet: $+\infty$
rechterlimiet $+\infty$

$$= \lim_{x \overset{<}{\to} 1} 0{,}22^{+\infty} = 0$$

(zie eigenschappen)

$$= \lim_{x \overset{>}{\to} 1} 0{,}22^{+\infty} = +\infty$$

f) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2+x+1}}{x+1}$ $\left(\sqrt{(-\infty)^2+(-\infty)} \text{ is onbepaald}\right)$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{\sqrt{x^2 \cdot \left(1+\frac{x}{x^2}+\frac{1}{x^2}\right)}}{x+1} = \lim_{x \to -\infty} \frac{-x \cdot \sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{x+1}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{-\cancel{x} \sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}{\cancel{x} \cdot \left(1+\frac{1}{x}\right)} = \lim_{x \to -\infty} \frac{-\sqrt{1+\cancel{\frac{1}{-\infty}}+\cancel{\frac{1}{(-\infty)^2}}}}{1+\cancel{\frac{1}{-\infty}}} = -\frac{\sqrt{1}}{1} = -1$$

g) $\lim\limits_{x \to -1} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$

$$\Leftrightarrow \lim_{x \to -1} \frac{\cancel{\sqrt{1+(-1)}}-\sqrt{1-(-1)}}{-1} = \frac{-\sqrt{2}}{-1} = \sqrt{2}$$

! $\sqrt{2}$ is rechterlimiet want linkerlimiet bestaat niet!!
↳ $\sqrt{1+x}$ met voorwaarde $(1+x) \geq 0$ dus $x \geq -1$

h) $\lim\limits_{x \to -1} \dfrac{x+1}{\sqrt{x^2+3}-2}$ (onbepaald, $\frac{0}{0}$)

$$\Leftrightarrow \lim_{x \to -1} \frac{x+1}{\sqrt{x^2+3}-2} \cdot \frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}$$

$$\Leftrightarrow \lim_{x\to -1} \frac{(x+1).\sqrt{x^2+3}+2}{x^2+3-4} = \lim_{x\to -1} \frac{(x+1).\sqrt{x^2+3}+2}{x^2-1}$$

$$\Leftrightarrow \lim_{x\to -1} \frac{\cancel{(x+1)}\sqrt{x^2+3}+2}{\cancel{(x+1)}(x-1)} = \lim_{x\to -1} \frac{\sqrt{(-1)^2+3}+2}{(-1-1)} = \frac{4}{-2} = -2$$

i. $\lim_{x\to 4}\left(\frac{1}{x-4} - \frac{15+x}{x^2-16}\right)$

$$= \lim_{x\to 4} \frac{1}{(x-4)} - \frac{15+x}{(x-4).(x+4)} \Rightarrow \frac{(x+4)}{(x-4).(x+4)} - \frac{15+x}{(x-4).(x+4)}$$

$$= \lim_{x\to 4} \frac{\cancel{x}+4-15-\cancel{x}}{(x+4).(x-4)} = \lim_{x\to 4} \frac{-11}{(x+4).(x-4)}$$

$\Downarrow$

		4	
$x-4$	$-$	0	$+$

linkerlimiet $-\infty$
rechterlimiet $+\infty$

j. $\lim_{x\to -2} \frac{1}{x^2+4x+4}$

$$\Leftrightarrow \lim_{x\to -2} \frac{1}{(-2)^2+4.(-2)+4} = \frac{1}{0}$$

$\Downarrow$ enig nulpunt, $D = 4^2-4.1.4=0$

		-2	
x^2+4x+4	$+$	0	$+$

$\begin{cases} \text{linkerlimiet } +\infty \\ \text{rechterlimiet } +\infty \end{cases} \Rightarrow \lim f(x) = +\infty$

k. $\lim_{x\to -\infty} \sqrt[3]{27x^3+9x} - x$

$$\Leftrightarrow \lim_{x\to -\infty} \sqrt[3]{27x^3+9x} - x \cdot \frac{\sqrt{27x^3+9x}+x}{\sqrt{27x^3+9x}+x}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{\left(\sqrt[3]{27x^3+9x}\right)^2 - x^2}{\sqrt{27x^3+9x}+x}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{\left(\sqrt[3]{27x^3 \cdot \left(1+\frac{\cancel{9x}1}{\cancel{27x^3}3x^2}\right)}\right)^2 - x^2}{\sqrt[3]{27x^3 \cdot \left(1+\frac{\cancel{9x}1}{\cancel{27x^3}3x^2}\right)}+x}$$

$$= \lim_{x \to -\infty} \frac{-9x^2 \cdot \left[\sqrt[3]{1+\frac{1}{3x^2}}\right]^2 - x^2}{-3x \cdot \sqrt{1+\frac{1}{3x^2}}+x}$$

$$= \lim_{x \to -\infty} \frac{x^{\cancel{2}} \cdot \left(-9 \cdot \left[\sqrt{1+\frac{1}{3x^2}}\right]^2 - 1\right)}{\cancel{x} \cdot \left(-3 \cdot \sqrt{1+\frac{1}{3x^2}}+1\right)}$$

$$= \lim_{x \to -\infty} \frac{\left(-9 \left[\sqrt[3]{1+\cancel{\frac{1}{3(-\infty)^2}}}\right]^2 - 1\right) \cdot -\infty}{-3 \cdot \sqrt{1+\cancel{\frac{1}{3(-\infty)^2}}}+1} = \frac{-\infty \cdot -8}{-2} = -\infty$$

11. $$\lim_{x \to +\infty} \sqrt{x^2-5x} - x$$

$$\Leftrightarrow \lim_{x \to +\infty} \sqrt{x^2-5x} - x \cdot \frac{\sqrt{x^2-5x}+x}{\sqrt{x^2-5x}+x}$$

$$\Leftrightarrow \lim_{x \to +\infty} \frac{\cancel{x^2}-5x-\cancel{x^2}}{\sqrt{x^2-5x}+x} = \lim_{x \to +\infty} \frac{-5x}{\sqrt{x^2 \cdot \left(1-\frac{5\cancel{x}}{x^{\cancel{2}}}\right)}+x}$$

$$\Leftrightarrow \lim_{x \to +\infty} \frac{-5x}{x \cdot \sqrt{1-\frac{5}{x}}+x} = \lim_{x \to +\infty} \frac{-5\cancel{x}}{\cancel{x} \cdot \left(\sqrt{1-\frac{5}{x}}+1\right)}$$

$$\Leftrightarrow \lim_{x \to +\infty} \frac{-5}{\sqrt{1-\cancel{\frac{5}{+\infty}}}+1} = \frac{-5}{2}$$

m. $\lim\limits_{x \to 1} \frac{x^3+4x-5}{x^2-1}$

$\Leftrightarrow \lim\limits_{x \to 1} \frac{x^3+4x-5}{(x-1).(x+1)}$

<u>horner</u>

	1	0	4	-5
1	↓	1	1	5
	1	1	5	0

$\rightsquigarrow (x-1).(x^2+x+5)$

$\Leftrightarrow \lim\limits_{x \to 1} \frac{\cancel{(x-1)}.(x^2+x+5)}{\cancel{(x-1)}(x+1)} = \lim\limits_{x \to 1} \frac{1^2+1+5}{1+1} = \frac{7}{2}$

n. $\lim\limits_{x \to 5} \frac{\frac{1}{(x-5)^2}}{\frac{1}{(x^2-25)}}$

$\Leftrightarrow \lim\limits_{x \to 5} \frac{(x^2-25)}{(x-5)^2} = \lim\limits_{x \to 5} \frac{\cancel{(x-5)}(x+5)}{(x-5).\cancel{(x-5)}} = \lim\limits_{x \to 5} \frac{10}{0}$

$\Downarrow$

		5	
$x-5$	$-$	0	$+$

linkerlimiet $-\infty$
rechterlimiet $+\infty$

o. $\lim\limits_{x \to -2} \frac{-2x-4}{x^3+2x^2}$

$\Leftrightarrow \lim\limits_{x \to -2} \frac{-2.(x+2)}{x^3+2x^2}$

<u>horner</u>

	1	2	0	0
-2	1	-2		
	1	0	0	0

$\rightsquigarrow (x+2).x^2$

$$= \lim_{x \to -2} \frac{-2\,(\cancel{x+2})}{x^2 \cdot (\cancel{x+2})} = \lim_{x \to -2} \frac{-2}{(-2)^2} = \frac{-2}{4} = -\frac{1}{2}$$

11 $\lim_{x \to -\infty} x + \sqrt{x^2+9}$

$$\Leftrightarrow \lim_{x \to -\infty} x + \sqrt{x^2+9} \cdot \frac{x - \sqrt{x^2+9}}{x - \sqrt{x^2+9}}$$

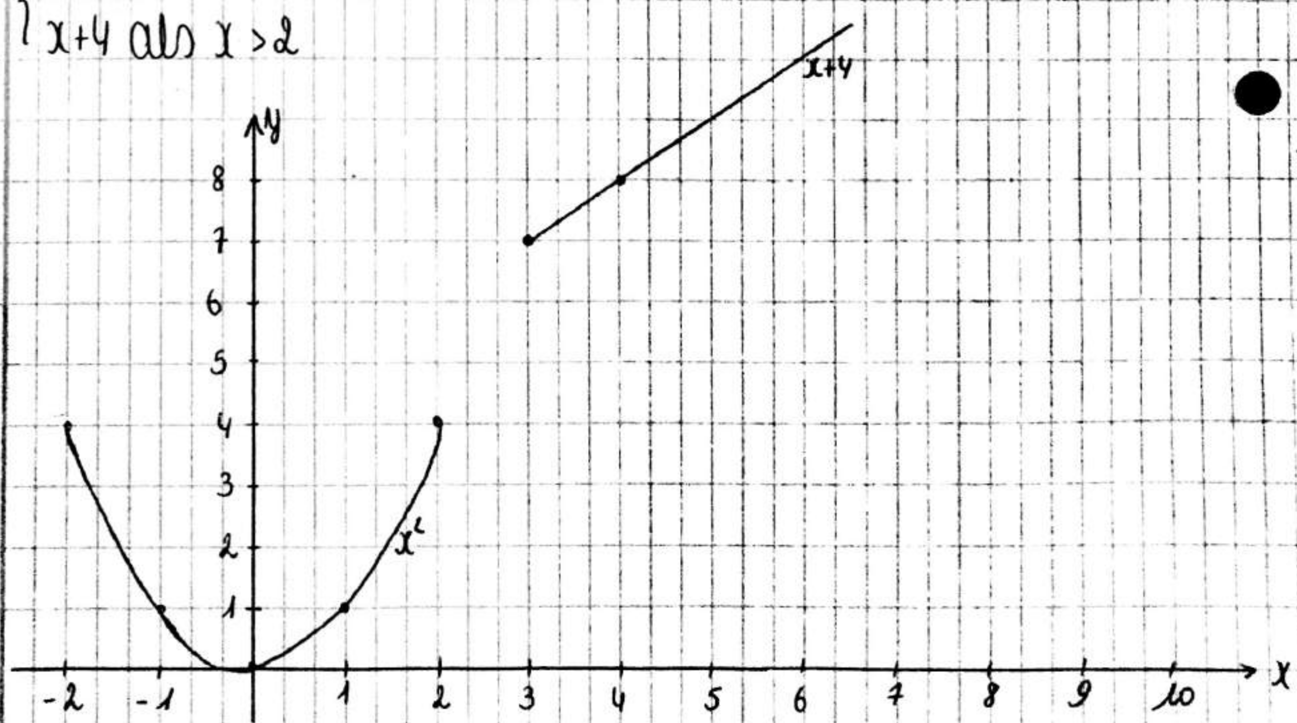
$$\Leftrightarrow \lim_{x \to -\infty} \frac{x^2 - x^2 - 9}{x - \sqrt{x^2+9}} = \lim_{x \to -\infty} \frac{-9}{x - \sqrt{x^2 \cdot (1 + \frac{9}{x^2})}}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{-9}{x + x \cdot \sqrt{1+\frac{9}{x^2}}} = \lim_{x \to -\infty} \frac{-9}{x \cdot (1 + \sqrt{1+\frac{9}{x^2}})}$$

$$\Leftrightarrow \lim_{x \to -\infty} \frac{-9}{-\infty \cdot (1 + \sqrt{1+\frac{9}{\cancel{(-\infty)^2}}})} = \frac{-9}{-\infty} = 0$$

OPGAVE 10

$$\begin{cases} x^2 & \text{als } x \le 2 \\ x+4 & \text{als } x > 2 \end{cases}$$



$\lim_{x \overset{>}{\to} 2} x+4 = 2+4 = 6$ → niet verschillend f(2) dus essentieel

$\lim_{x \overset{<}{\to} 2} x^2 = 2^2 = 4$

OPGAVE 9 a. $f(x) = \frac{4x^2-2x+7}{x-3}$

- VA:

$\boxed{x=3}$ potentiële asymptoot

$\lim_{x \to 3} \frac{4x^2-2x+7}{x-3} = \frac{36-6+7}{0} = \frac{37}{0}$

		3	
$x-3$	$-$	0	$+$

LL: $-\infty$

RL: $+\infty$

$\boxed{x=3 \text{ is VA!}}$

- HA/SA:

$\lim_{x \to +\infty} \left(\frac{4x^2-2x+7}{x-3}\right) \cdot \frac{1}{x} = \lim_{x \to +\infty} \frac{4x^2-2x+7}{x^2-3x}$

$= \lim_{x \to +\infty} \frac{4\cancel{x^2} \cdot \left(1 - \frac{2x}{4x^2} + \frac{7}{4x^2}\right)}{\cancel{x^2} \cdot \left(1 - \frac{3x}{x^2}\right)}$

$= \lim_{x \to +\infty} \frac{\left(1 - \cancel{\frac{2}{4 \cdot +\infty}} + \cancel{\frac{7}{4 \cdot (+\infty)^2}}\right) \cdot 4}{\left(1 - \cancel{\frac{3}{+\infty}}\right)} = 4$ → SA!

ZELFDE VOOR $-\infty$!

$\lim_{x \to +\infty} \left(\frac{4x^2-2x+7}{x-3}\right) - 4x \Leftrightarrow \lim \frac{\cancel{4x^2} - 2x + 7 - \cancel{4x^2} + 12x}{x-3}$

$\lim_{x \to +\infty} \frac{10x+7}{x-3} \Leftrightarrow \lim_{x \to +\infty} \frac{\cancel{x} \cdot \left(10 + \frac{7}{x}\right)}{\cancel{x} \cdot \left(1 - \frac{3}{x}\right)} = 10$ → $4x + 10 = SA$

b. $f(x) = \dfrac{x^3 - 5x^2 + 3x + 1}{x^2}$

- VA:

0 is potentiële asymptoot

$$\lim_{x \to 0} \frac{x^3 - 5x^2 + 3x + 1}{x^2} = \frac{0^3 - 5.0^2 + 3.0 + 1}{0^2} = \frac{1}{0}$$

		0	
x^2	+	0	+

LL: $+\infty$
RL: $+\infty$

VA: $x = 0$

- HA/SA

$$\lim_{x \to +\infty} \frac{x^3 - 5x^2 + 3x + 1}{x^2} \cdot \frac{1}{x} = \lim_{x \to +\infty} \frac{x^3 - 5x^2 + 3x + 1}{x^3}$$

$$\Leftrightarrow \lim_{x \to +\infty} \frac{\cancel{x^3} \cdot \left(1 - \frac{5\cancel{x^2}}{x^{\cancel{3}}} + \frac{3\cancel{x}}{x^{\cancel{3}2}} + \frac{1}{x^3}\right)}{\cancel{x^3}} = 1 \rightsquigarrow \text{SA}.$$

IDEM VOOR $-\infty$!

$$\lim_{x \to +\infty} \frac{x^3 - 5x^2 + 3x + 1}{x^2} - x = \lim_{x \to +\infty} \frac{\cancel{x^3} - 5x^2 + 3x + 1 - \cancel{x^3}}{x^2}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^2} \cdot \left(-5 + \frac{3\cancel{x}}{x^{\cancel{2}}} + \frac{1}{x^2}\right)}{\cancel{x^2}} = -5 \rightsquigarrow \boxed{\text{SA} = x - 5}$$

c. $f(x) = \sqrt{x^2 + x}$

- VA:

geen VA!

- HA/SA

$$\lim_{x \to +\infty} \frac{\sqrt{x^2 + x}}{x} = \lim_{x \to +\infty} \frac{\sqrt{x^2 \cdot \left(1 + \frac{x}{x^2}\right)}}{x}$$

hierdoor 2 asymptoten!!
vanaf hier opsplitsen $-\infty$ en $+\infty$

$|x| = x$ naar $+\infty$
$|x| = -x$ naar $-\infty$

x^2 "verdoezelt" negatieve getallen!

$$\lim_{x\to+\infty} \frac{|x|\cdot\sqrt{1+\frac{1}{x}}}{x} = \lim_{x\to+\infty} \frac{\cancel{x}\cdot\sqrt{1+\frac{1}{x}}}{\cancel{x}} = 1 = \text{SA!}$$

idem berekening ✓

$$\lim_{x\to-\infty} \frac{-\cancel{x}\cdot\sqrt{1+\frac{1}{x}}}{\cancel{x}} = -1 = \text{SA!}$$

ASYM. NAAR $+\infty$

$$\lim_{x\to+\infty} \sqrt{x^2+x} - x = \lim_{x\to+\infty} \sqrt{x^2+x} - x \cdot \frac{\sqrt{x^2+x}+x}{\sqrt{x^2+x}+x}$$

$$= \lim_{x\to+\infty} \frac{\cancel{x^2}+x-\cancel{x^2}}{\sqrt{x^2+x}+x} = \lim_{x\to+\infty} \frac{x}{\sqrt{x^2\cdot\left(1+\frac{\cancel{x}}{x^{\cancel{2}}}\right)}+x}$$

$$= \lim_{x\to+\infty} \frac{x}{|x|\cdot\sqrt{1+\frac{1}{x}}+x} = \lim_{x\to+\infty} \frac{\cancel{x}}{\cancel{x}\cdot\left(\sqrt{1+\frac{1}{x}}+1\right)} = \frac{1}{2}$$

✓ idem berekening naar toegevoegde was $\sqrt{x^2+x}-x$

ASYMP. NAAR $-\infty$

$$\lim_{x\to-\infty} \frac{x}{|x|\cdot\sqrt{1+\frac{1}{x}}-x} = \lim_{x\to-\infty} \frac{x}{-x\cdot\sqrt{1+\frac{1}{x}}-x}$$

$$= \lim_{x\to-\infty} \frac{-\cancel{x}}{\cancel{x}\cdot\left(\sqrt{1+\frac{1}{x}}+1\right)} = -\frac{1}{2}$$

$$\boxed{\text{SA naar } +\infty\text{: } x+\frac{1}{2}}$$

$$\boxed{\text{SA naar } -\infty\text{: } -x-\frac{1}{2}}$$

OPGAVE 11 a. raaklijn $\sqrt{x}$ in $(4,2)$

$y-2=\frac{1}{4}\cdot(x-4)$

$f'(x)=\frac{1}{2\sqrt{x}}=\frac{1}{2\sqrt{4}}=\frac{1}{4}$

$\Leftrightarrow y-2=\frac{1}{4}x-1$

$\Leftrightarrow \boxed{y=\frac{1}{4}x+1}$

b. $y-f(-1)=f'(x).(x+1)$

$f'(x)=2^x\ln 2=2^{-1}\ln 2=\frac{\ln 2}{2}$

$\Leftrightarrow y-\frac{1}{2}=\frac{\ln 2}{2}.(x+1)$

$\Leftrightarrow y-\frac{1}{2}=\frac{\ln 2}{2}x+\frac{\ln 2}{2}$

$\Leftrightarrow y=\frac{\ln 2}{2}x+\frac{\ln 2}{2}+\frac{1}{2}$

$\Leftrightarrow \boxed{y=\frac{\ln 2}{2}x+\frac{1}{2}.(\ln 2+1)}$

c. $y-f(3)=f'(x).(x-3)$

$f'(x)=\frac{(x-2)^2-2x.(x-2)}{(x-2)^4}=\frac{1-6}{1}$

$\Leftrightarrow y-3=-5.(x-3)$

$\Leftrightarrow y-3=-5x+15$

$\Leftrightarrow \boxed{y=-5x+18}$

OPGAVE 14 a. $f(x)=x^{(x^2)}$

$f(x)=e^{\ln x.x^2}$

$\Leftrightarrow f'(x)=e^{\ln x.x^2}.(\ln x.x^2)'$

$\Leftrightarrow f'(x)=e^{\ln x.x^2}.((\ln x)'.x^2+\ln x.(x^2)')$

$\Leftrightarrow f'(x)=e^{\ln x.x^2}.(\frac{x^{\cancel{2}}}{\cancel{x}}+\ln x.2x)$

$\Leftrightarrow f'(x)=x^{x^2}.(x+2x.\ln x)$

$$\Leftrightarrow f'(x) = x^{x^2} \cdot x \cdot (1 + 2\ln x)$$

$$\Leftrightarrow \boxed{f'(x) = x^{x^2+1} \cdot (1 + 2\ln x)}$$

b. $f(x) = (\tan x)^{\sqrt[3]{x}}$

$$f(x) = e^{\ln(\tan x) \cdot \sqrt[3]{x}}$$

$$\Leftrightarrow f'(x) = e^{\ln(\tan x) \cdot \sqrt[3]{x}} \cdot (\ln(\tan x) \cdot \sqrt[3]{x})'$$

$$\Leftrightarrow f'(x) = e^{\ln(\tan x) \cdot \sqrt[3]{x}} \cdot ((\ln \tan x)' \cdot \sqrt[3]{x} + \ln|\tan x| \cdot (\sqrt[3]{x})')$$

$$\Leftrightarrow f'(x) = e^{\ln(\tan x) \cdot \sqrt[3]{x}} \left[\frac{1}{\tan x} \cdot \frac{1}{\cos^2 x} \cdot \sqrt[3]{x} + \ln|\tan x| \cdot \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}\right]$$

$$\Leftrightarrow f'(x) = e^{\ln(\tan x) \cdot \sqrt[3]{x}} \cdot \left[\frac{\cancel{\cos x} \cdot \sqrt[3]{x}}{\sin x \cdot \cos^{\cancel{2}} x} + \frac{\ln|\tan x|}{3\sqrt[3]{x^2}}\right]$$

$$f'(x) = e^{\ln(\tan x) \cdot \sqrt[3]{x}} \cdot \left[\frac{\sqrt[3]{x} \cdot \sqrt[3]{x^2}}{\sin x \cdot \cos x \cdot \sqrt[3]{x^2}} + \frac{\ln|\tan x| \cdot \cancel{\sqrt[3]{x^2}}}{3\cancel{\sqrt[3]{x^2}} \cdot \sqrt[3]{x^2}}\right]$$

$$f'(x) = \frac{\tan x^{\sqrt[3]{x}}}{\sqrt[3]{x^2}} \cdot \left[\frac{x}{\sin x \cdot \cos x} + \frac{\ln|\tan x|}{3}\right]$$

$$\boxed{f'(x) = \frac{\tan x^{\sqrt[3]{x}}}{\sqrt[3]{x^2}} \cdot \left[\frac{3x}{\sin x \cdot \cos x} + \ln|\tan x|\right]}$$

c. $f(x) = (\cos(3x+1))^{\sin x}$

$$f(x) = e^{\ln \cos x(3x+1) \cdot \sin x}$$

$$\Leftrightarrow f'(x) = e^{\ln|\cos x(3x+1)| \cdot \sin x} \cdot (\ln|\cos x(3x+1)| \cdot \sin x)'$$

$$\Leftrightarrow f'(x) = \cos(3x+1)^{\sin x} \cdot ((\ln|\cos(3x+1)|)' \cdot \sin x + \ln|\cos(3x+1)| \cdot (\sin x)')$$

$$\Leftrightarrow f'(x) = \cos(3x+1)^{\sin x} \left[\frac{1}{\cos(3x+1)} \cdot -\sin(3x+1) \cdot 3 \cdot \sin x + \ln|\cos(3x+1)| \cdot \cos x\right]$$

$$\Leftrightarrow f'(x) = \cos(3x+1)^{\sin x} \cdot \left[\frac{-3\sin(3x+1) \cdot \sin x}{\cos(3x+1)} + \ln|\cos(3x+1)| \cdot \cos x\right]$$

$$\Leftrightarrow \boxed{f'(x) = \cos(3x+1)^{\sin x} \cdot (-3\tan(3x+1) \cdot \sin x + \ln|\cos(3x+1)| \cdot \cos x)}$$

d. $f(x) = \left(\frac{x^2+3}{6x+2}\right)^x$

$f(x) = e^{\ln\left(\frac{x^2+3}{6x+2}\right)\cdot x}$

$\Leftrightarrow f'(x) = \left(\frac{x^2+3}{6x+2}\right)^x \cdot \left(\left(\ln\left(\frac{x^2+3}{6x+2}\right)\right)' \cdot x + \ln\left(\frac{x^2+3}{6x+2}\right)\cdot x'\right)$

$f'(x) = \left(\frac{x^2+3}{6x+2}\right)^x \cdot \left(\frac{1}{\left(\frac{x^2+3}{6x+2}\right)} \cdot \frac{(x^2+3)'(6x+2)-(x^2+3)(6x+2)'}{(6x+2)^2} + \ln\left(\frac{x^2+3}{6x+2}\right)\cdot x'\right)$

$f'(x) = \left(\frac{x^2+3}{6x+2}\right)^x \cdot \left(\frac{(6x+2)}{(x^2+3)} \cdot \frac{2x\cdot(6x+2)-(x^2+3)\cdot 6}{(6x+2)^2} + \ln\left(\frac{x^2+3}{6x+2}\right)\right)$

$f'(x) = \left(\frac{x^2+3}{6x+2}\right)^x \cdot \left(\left(\frac{\cancel{(6x+2)}2x - 6\cdot(x^2+3)\cancel{(6x+2)}}{(x^2+3)\cdot(6x+2)^{\cancel{2}}}\right)\cdot x + \ln\left(\frac{x^2+3}{6x+2}\right)\right)$

$f'(x) = \left(\frac{x^2+3}{6x+2}\right)^x \cdot \left(\frac{\cancel{(6x+2)}2x}{(x^2+3)\cancel{(6x+2)}} - \frac{6\cancel{(x^2+3)}}{\cancel{(x^2+3)}(6x+2)}\cdot x + \ln\left(\frac{x^2+3}{6x+2}\right)\right)$

$$\boxed{f'(x) = \left(\frac{x^2+3}{6x+2}\right)^x \cdot \left(\left[\frac{2x}{(x^2+3)} - \frac{6}{(6x+2)}\right]\cdot x + \ln\left(\frac{x^2+3}{6x+2}\right)\right)}$$

OPGAVE 12

$(\sqrt{4-3x^2})' = 3$

$\frac{-\cancel{6}^{3}x}{\cancel{2}\cdot\sqrt{4-3x^2}} = 3$

$\Leftrightarrow -\cancel{3}x = \sqrt{4-3x^2}\cdot\cancel{3}$ $\quad ^2$ zorgt voor bijkomende voorwaarde!

$\Leftrightarrow x^2 = 4-3x^2$

$\Leftrightarrow \cancel{4}x^2 = \cancel{4}1$

$\Leftrightarrow x = \pm\sqrt{1}$ $\rightsquigarrow$ x moet kleiner zijn dan 0!

$\cancel{x=1}$ of $x = -1$

$y - f(-1) = 3\cdot(x+1)$

$y - 1 = 3x+3$

$\boxed{y = 3x+4}$

OPGAVE 13 $f(x) = x.(1-x)^2$

$f'(x) = (x)'.(1-x)^2 + x.((1-x)^2)'$

$f'(x) = (1-x)^2 + 2x.(1-x).(-1) \Leftrightarrow f'(x) = (1-x)^2 - 2x.(1-x)$

$0 = (1-x)^2 - 2x + 2x^2$

$\Leftrightarrow 0 = 1 - 2x + x^2 - 2x + 2x^2 = 3x^2 - 4x + 1$

$D = (-4)^2 - 4.3.1 = 16 - 12 = 4$

$\frac{4 \pm \sqrt{4}}{6} = 1$ of $1/3$

$f''(x) = -2.(1-x) - 2 + 2x$

$f''(x) = -2 + 2x - 2 + 2x = 4x - 4$

als $1 \Rightarrow 0$ lokaal minimum

$1/3 \Rightarrow 8/27$ lokaal maximum

punten interval? grootste waarde interval!

$1/2 \Rightarrow 1/8$

$2 \Rightarrow 2.(1-2)^2 = 2$ globaal maximum

CONCLUSIE: minimum (1,0)

maximum (2,2)

OPGAVE 15 a. $y = \frac{e^{x^2}.x \ln x}{\sqrt{x+3}}$

$\mathcal{E}_{e^{x^2}} + \mathcal{E}_x + \mathcal{E}_{\ln x} - \mathcal{E}_{\sqrt{x+3}}$

$= \left[\frac{\cancel{e^{x^2}}.2x}{\cancel{e^{x^2}}}\right].x + \left[\frac{1}{\cancel{x}}\right]\cancel{x} + \left[\frac{1}{\cancel{x}.\ln x}\right]\cancel{x} - \left[\frac{1}{2\sqrt{x+3}.\sqrt{x+3}}\right].x$

$= \boxed{2x^2 + 1 + \frac{1}{\ln x} - \frac{x}{2.(x+3)}}$

b. $y = \frac{3x^5.\sin^3 x}{e^{2x}}$

$\mathcal{E}_{3x^5} + \mathcal{E}_{\sin^3 x} - \mathcal{E}_{e^{2x}}$

$$= \left[\frac{\cancel{15x^4}}{\cancel{3x^5}}\right]\cdot x + \left[\frac{3\cdot\cancel{\sin^2 x}\cdot\cos x}{\cancel{\sin^3 x}}\right]\cdot x - \left[\frac{\cancel{e^{2x}}\cdot 2}{\cancel{e^{2x}}}\right]\cdot x$$

$$= 5 + \frac{3x\cdot\cos x}{\sin x} - 2x$$

$$= \boxed{5 + 3x\cdot\cot x - 2x}$$

c. $y = \dfrac{7x^3\cdot e^{5x}\cdot\sin x}{3x^2+x-1}$

$$\mathcal{E}_{7x^3} + \mathcal{E}_{e^{5x}} + \mathcal{E}_{\sin x} - (\mathcal{E}_{3x^2+x-1})$$

$$= \left[\frac{21\cancel{x^2}}{7x^{\cancel{3}}}\right]\cdot x + \left[\frac{\cancel{e^{5x}}\cdot 5}{\cancel{e^{5x}}}\right]\cdot x + \left[\frac{\cos x}{\sin x}\right]\cdot x - \left[\frac{6x+1}{3x^2+x-1}\right]\cdot x$$

$$= 3 + 5x + \cot x\cdot x - \frac{6x^2+x}{3x^2+x-1}$$

$$= \boxed{3+5x+x\cdot\cot x - \frac{x(6x+1)}{3x^2+x-1}}$$

OPGAVE 16

a. $\lim\limits_{x\to0} \dfrac{2^{4x}-2^{-4x}}{x^4} \quad \left(=\frac{0}{0}\right)$

$$\overset{H}{=} \frac{4\ln2\cdot e^{4x\ln2}+4\ln2\cdot e^{-4x\ln2}}{4x^3}$$

$$= \frac{4\ln2}{4}\cdot\lim_{x\to0}\frac{e^{4x\cdot\ln2}+e^{-4x\ln2}}{x^3}$$

		0	
x^3	$-$	0	$+$

linkerlimiet: $-\infty$
rechterlimiet: $+\infty$

b. $\lim\limits_{x\to+\infty} x\, e^{-x^2}$

$\lim\limits_{x\to+\infty} \dfrac{x}{e^{x^2}} \quad \left(=\frac{+\infty}{+\infty}\right)$

$$\overset{H}{=} \frac{1}{e^{x^2}\cdot 2x} = \frac{1}{+\infty} = 0 \quad \text{(in principe linkerlimiet!)}$$

c. $\lim\limits_{x\to 0} x \cdot e^{\frac{1}{x}}$ (onbepaald!)

$$= \lim_{x\to 0} \frac{e^{\frac{1}{x}}}{1/x} \overset{H}{=} \lim_{x\to 0} \frac{\cancel{-\frac{1}{x^2}}\cdot e^{1/x}}{\cancel{-1/x^2}}$$

		0	
$1/x$	$-\infty$	|	$+\infty$

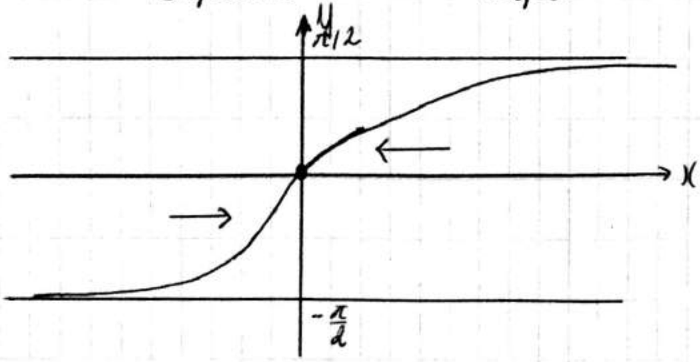
$\boxed{\begin{array}{l}\text{linkerlimiet: } e^{-\infty} = 0 \\ \text{rechterlimiet: } e^{+\infty} = +\infty\end{array}}$

d. $\lim\limits_{x\to +\infty} \frac{x}{\ln(3x)+e^{7x}} \left(= \frac{+\infty}{+\infty}\right)$

$$\overset{H}{=} \lim_{x\to +\infty} \frac{1}{\frac{1}{3x}\cdot 1 + 7\cdot e^{7x}} = \frac{1}{+\infty} = 0 \quad \text{(in principe linkerlimiet!)}$$

e. $\lim\limits_{x\to 2} \frac{e^{x} - e^{x/2}}{\text{Bgtan}(x-2)}$

$$= \lim_{x\to 2} \frac{e^2 - e^1}{\text{Bgtan}\, 0} = \lim_{x\to 2} \frac{e^2 - e}{\text{Bgtan}\, 0}$$



$\boxed{\begin{array}{l}\text{linkerlimiet: } -\infty \\ \text{rechterlimiet: } +\infty\end{array}}$

OPGAVE 17 a. $f(x) = x \cdot e^{x}$

VA? geen noemer, dus geen VA

HA/SA? $\lim\limits_{x\to \pm\infty} \frac{\cancel{x}\cdot e^{x}}{\cancel{x}}$ → $\lim\limits_{x\to +\infty} e^{x} = +\infty$

→ $\lim\limits_{x\to -\infty} e^{x} = 0$ → $\boxed{\text{HA naar } -\infty}$

$\lim_{x \to -\infty} x \cdot e^{x}$ (= onbepaald!)

$= \lim_{x \to -\infty} \frac{x}{e^{-x}} \overset{H}{=} \lim_{x \to -\infty} \frac{1}{-e^{-x}} = \frac{1}{-\infty} = 0$

$\boxed{HA: y = 0}$ (naar $-\infty$)

b. $f(x) = e^{\frac{1}{x}} - 1$

VA: $x \overset{?}{=} 0$ $\qquad x \underset{>}{\to} 0 \quad \frac{1}{x} = +\infty$

$\qquad\qquad\qquad x \underset{<}{\to} 0 \quad \frac{1}{x} = -\infty$

$\lim_{x \underset{>}{\to} 0} e^{+\infty} - 1 = +\infty$ $\Rightarrow$ VA!

$\lim_{x \underset{<}{\to} 0} e^{-\infty} - 1 = -1$

HA/SA?

$\lim_{x \to +\infty} \frac{e^{1/x} - 1}{x} = \frac{0}{+\infty} = 0$

$\lim_{x \to -\infty} \frac{e^{1/x} - 1}{x} = \frac{-1}{-\infty} = 0$

$\lim_{x \to +\infty} e^{1/x} - 1 = e^{0} - 1 = 1 - 1 = 0$

$$\boxed{\begin{array}{l} \text{VA naar } -\infty\text{: } x = 0 \\ \text{HA: } y = 0 \end{array}}$$

c. $f(x) = \frac{x^3 + x^2 - 2}{1 - x^2}$

VA: $x \overset{?}{=} 1$ of $x \overset{?}{=} -1$

$\lim_{x \to 1} \frac{x^3 + x^2 - 2}{1 - x^2} \left(= \frac{0}{0}\right) \overset{H}{=} \lim_{x \to 1} \frac{3x^2 + 2x}{-2x} = \frac{3+2}{-2} = \frac{5}{-2}$

$\lim_{x \to -1} \frac{x^3 + x^2 - 2}{1 - x^2} = \frac{-1+1-2}{0} = \frac{-2}{0}$ linkerlimiet $+\infty$

rechterlimiet $+\infty$

MA|SA?

$$\lim_{x\to\pm\infty} \frac{x^3+x^2-2}{x.(1-x^2)} = \lim_{x\to\pm\infty} \frac{x^3+x^2-2}{x-x^3}$$

$$\lim_{x\to\pm\infty} \frac{\cancel{x^3}.\left(1+\frac{x^2}{x^3}-\frac{2}{x^3}\right)}{\cancel{x^3}.\left(\frac{x}{x^3}-1\right)} = -1$$

$$\lim_{x\to+\infty} \frac{x^3+x^2-2}{(1-x^2)} + x$$

$$\lim_{x\to+\infty} \frac{\cancel{x^3}+x^2-2+x-\cancel{x^3}}{(1-x^2)} = \lim_{x\to+\infty} \frac{x^2+x-2}{(1-x^2)}$$

$$= \lim_{x\to+\infty} \frac{\cancel{x^2}.\left(1+\frac{x}{x^2}-\frac{2}{x^2}\right)}{\cancel{x^2}.\left(\frac{1}{x^2}-1\right)} = -1$$

$$\boxed{\begin{array}{l} VA: x=-1 \\ SA: y=-x-1 \end{array}}$$

OPGAVE 18

a. $E_y = \frac{-4p^2}{200-2p^2} = \frac{-2p^2}{100-p^2}$

$$E(6) = \frac{-2.6^2}{100-6^2} = -\frac{72}{64}.0{,}25 = \frac{-72}{256}\,\% = \frac{-9}{32}\,\%$$

b. $E(6) = -\frac{72}{64} = -\frac{9}{8} < -1$ dus elastisch waardoor prijzen ↓

OPGAVE 19

$$f(x) = x.e^x$$
$$f'(x) = e^x + x.e^x = e^x.(1+x)$$
$$f''(x) = e^x(1+x) + e^x = e^x.(x+2)$$

$f'(x) = 0 \Leftrightarrow e^x.(x+1) = 0 \Leftrightarrow \boxed{x = -1}$

$f''(-1) = e^{-1}.(-1+2) \neq 0$ ↙ maximum of minimum!

$f''(x) = 0 \Leftrightarrow e^x (x+2) = 0 \Leftrightarrow \boxed{x = -2}$

$f'(-2) = -e^{-2} = -\frac{1}{e^2}$ = buigpunt!

$y - y_1 = f'(x_1).(x - x_1)$

$y - f(-2) = -\frac{1}{e^2}.(x+2)$

$y = -e^{-2}x - 2.e^{-2} - 2.e^{-2}$

$\boxed{y = -e^{-2x} - 4.e^{-2}}$

buigpunt? ↗ $f'(x) = 0$ en $f''(x) = 0$
↘ $f'(x) \neq 0$ en $f''(x) = 0$

OPGAVE 20 a. $\int \frac{\ln x}{\sqrt{x}}\, dx$

$f(x) = \ln x \longrightarrow f'(x) = 1/x$

$g(x) = 2.x^{1/2} \longleftarrow g'(x) = x^{-1/2}$

$= \ln x . 2\sqrt{x} - 2.\int \frac{\sqrt{x}}{x}\, dx$

$= \ln x . 2\sqrt{x} - 2\int x^{-1/2}\, dx$

$= \boxed{\ln x . 2\sqrt{x} - 4\sqrt{x} + C}$

b. $\int \frac{x}{\sqrt[3]{2x+2}}\, dx$

$2x+2 = u^3 \rightsquigarrow x = \frac{u^3 - 2}{2} = \frac{u^3}{2} - 1$

$dx = 3u^2\, du$

$= \int \frac{u^3 - 1}{2u}.(3u^2\, du)$

$$= \int \frac{3u^5\,du - 3u^2\,du}{2u}$$

$$= \int \frac{3u^5\,du}{2u} - \int \frac{3u^2}{2u}\,du$$

$$= \frac{3}{2} \cdot \int \frac{u^5}{u}\,du - \frac{3}{2} \cdot \int \frac{u^2}{u}\,du$$

$$= \frac{3}{2} \int u^4\,du - \frac{3}{2} \int u\,du$$

$$= \frac{3}{2} \cdot \frac{u^5}{5} - \frac{3}{2} \cdot \frac{u^2}{2} + C$$

$$= \frac{3}{10} \cdot u^5 - \frac{3}{4}\,u^2 + C$$

$$= \boxed{\frac{3}{10}\sqrt[3]{(2x+2)^5} - \frac{3}{4} \cdot \sqrt[3]{(2x+2)^2} + C}$$

c. $\int \frac{x^7}{\sqrt{x^4+3}}\,dx$

$$= \int x^7 \cdot (x^4+3)^{-1/2}\,dx$$

$x^4 + 3 = u$

$4x^3\,dx = du$

$x^3\,dx = 1/4\,du$

$$= \frac{1}{4} \int (u-3) \cdot (u)^{-1/2}\,du$$

$$= \frac{1}{4} \int u^{1/2}\,du - \frac{1}{4} \cdot \int 3u^{-1/2}\,du$$

$$= \frac{1}{4} \cdot \frac{u^{3/2}}{3} \cdot 2 - \frac{1}{4} \cdot u^{1/2} \cdot 3 \cdot 2 + C$$

$$= \frac{2}{12}\,u^{3/2} - \frac{6}{4} \cdot u^{1/2} + C$$

$$= \boxed{\frac{1}{6} \cdot \sqrt{(x^4+3)^3} - \frac{3}{2} \cdot \sqrt{(x^4+3)} + C}$$

d. $\int \frac{\arctan(\sqrt{x}+1)}{\sqrt{x}}\,dx$

$= \int \frac{\arctan(\sqrt{x}+1)}{\sqrt{x}}\,dx$

$\sqrt{x}+1 = u$

$\frac{1}{2\sqrt{x}} = du$

$\frac{1}{\sqrt{x}} = 2\,du$

$= 2 . \int \arctan u \, du$

$f(x) = 2 \arctan u \longrightarrow \frac{2}{1+u^2} = f'(x)$

$g(x) = u \longleftarrow 1 = g'(x)$

$= 2 \arctan u . u - 2\int \frac{1}{1+u^2} . u \, du$

$1+u^2 = v$

$2u = dv$

$u = 1/2 \, dv$

$= 2 \arctan u . u - \cancel{2} . \frac{1}{\cancel{2}} \int \frac{dv}{v}$

$= 2 \arctan u . u - \ln|v| + C$

$= 2 \arctan u . u - \ln|1+u^2| + C$

$= \boxed{2 \arctan(\sqrt{x}+1).(\sqrt{x}+1) - \ln|1+(\sqrt{x}+1)^2| + C}$

u. $\int \frac{\tan(\ln x)}{x}\,dx$

$\ln x = u$

$1/x = du$

$= \int \tan u \, du$

$= \int \frac{\sin u}{\cos u}\,du$

$\cos u = v$

$-\sin u = dv$

$\sin u = -dv$

$-\int \frac{dv}{v}$

$= -\ln|v| + C$

$= -\ln|\cos u| + C$

$= \boxed{-\ln|\cos(\ln x)| + C}$

4. $\int (x - 2x^2) \cdot \cos\left(\frac{x}{2}\right) dx$

$= \int x \cdot \cos\left(\frac{x}{2}\right) dx - 2\int x^2 \cdot \cos\left(\frac{x}{2}\right) dx$

$\frac{x}{2} = t$

$\downarrow \cdot 2$

$x = 2t$

$dx = 2dt$

$2\int 4t^2 \cos t \cdot 2dt$

$8\int t^2 \cdot \cos t \cdot 2dt$

$16\int t^2 \cos t\, dt$

$= 4\underbrace{\boxed{\int t \cdot \cos t\, dt}}_{A} - 16\underbrace{\boxed{\int t^2 \cdot \cos t\, dt}}_{B}$

berekenen B: $\int t^2 \cdot \cos t\, dt$

$f(x) = t^2 \longrightarrow f'(x) = 2t$

$g(x) = \sin t \longleftarrow g'(x) = \cos t$

$\overset{PI}{=} t^2 \cdot \sin t - 2 \cdot \int t \cdot \sin t$

$f(x) = t \longrightarrow f'(x) = 1$

$g(x) = -\cos t \longleftarrow g'(x) = \sin t$

$\overset{PI}{=} -t \cdot \cos t + \int \cos t\, dt$

$= -t \cdot \cos t + \sin t + C$

$\boxed{B} = t^2 \cdot \sin t - 2 \cdot (-t \cos t + \sin t) + C$

$= \boxed{t^2 \cdot \sin t + 2t \cos t - 2\sin t + C}$

berekenen A: $\int t \cdot \cos t \, dt$

$f(x) = t \qquad f'(x) = 1$

$g(x) = \sin t \qquad g'(x) = \cos t$

$\overset{PI}{=} \sin t \cdot t - \int \sin t \, dt$

$= \boxed{\sin t \cdot t + \cos t + C}$

totale integraal $= 4 \cdot (t \sin t + \cos t) - 16\,(t^2 \sin t + 2t \cos t - 2 \sin t) + C$

$= 4t \sin t + 4 \cos t - 16t^2 \sin t - 32\, t \cos t + 32 \sin t + C$

$= \cancel{4}2\, \frac{x}{\cancel{2}} \cdot \sin\left(\frac{x}{2}\right) + 4 \cos\left(\frac{x}{2}\right) - \frac{\cancel{16}}{4}\left(\frac{x}{\cancel{2}}\right)^2 \sin\left(\frac{x}{2}\right) - \frac{\cancel{32}}{16}\left(\frac{x}{\cancel{2}}\right) \cos\left(\frac{x}{2}\right) + 32 \sin\left(\frac{x}{2}\right) + C$

$= 2x \cdot \sin\left(\frac{x}{2}\right) + 4 \cos\left(\frac{x}{2}\right) - 4 \cdot \underline{x}^2 \sin\left(\frac{x}{2}\right) - 16x \cdot \cos\left(\frac{x}{2}\right) + 32 \sin\left(\frac{x}{2}\right) + C$

$= \boxed{\sin\left(\frac{x}{2}\right) \cdot (2x - 4x^2 + 32) + \cos\left(\frac{x}{2}\right) \cdot (4 - 16x) + C}$

g. $\int \cos x \cdot \sin x \cdot e^{\cos x} \, dx$

$\cos x = u$

$-\sin x = du$

$\sin x = -du$

$-\int u \cdot e^u \, du$

$f(x) = u \longrightarrow f'(x) = 1$

$g(x) = e^u \longleftarrow g'(x) = e^u$

$\overset{PI}{=} -u \cdot e^u + \int e^u \, du$

$= -u \cdot e^u + e^u + C$

$= -\cos x \cdot e^{\cos x} + e^{\cos x} + C$

$= \boxed{e^{\cos x} \cdot (-\cos x + 1) + C}$

h. $\int (x^3 + 1) \cdot \text{Bgsin}\,(x^4 + 4x)\, dx$

$x^4 + 4x = u$

$4x^3 + 4 = du$

$x^3 + 1 = \frac{1}{4} du$

$$= \frac{1}{4}\boxed{\int \arcsin u \, du}$$

$$f(x) = \arcsin u \longrightarrow f'(x) = \frac{1}{\sqrt{1-u^2}}$$

$$g(x) = u \longleftarrow g'(x) = 1$$

$$= \arcsin u \, . \, u - \int \frac{1}{\sqrt{1-u^2}} \, . \, u \, du$$

$$1 - u^2 = v$$
$$-2u \, dx = dv$$
$$u \, du = -\frac{1}{2} dv$$

$$= \arcsin u \, . \, u + \frac{1}{2} \int \frac{dv}{v}$$

$$= \arcsin u \, . \, u + \frac{1}{2} \, . \, 2 \, . \, \sqrt{v} + C$$

$$= \arcsin u \, . \, u + \sqrt{(1-u^2)} + C$$

$$= \arcsin(x^4+4x) \, . \, (x^4+4x) + \sqrt{(1-(x^4+4x)^2} + C$$

$$= \boxed{\frac{1}{4} \arcsin(x^4+4x) \, . \, (x^4+4x) + \sqrt{(1-(x^4+4x)^2} + C}$$

i. $\int \frac{1}{x^2-2x+1} \, dx$

$$= \int \frac{1}{(x-1)^2} \, dx$$

$$(x-1) = u$$
$$dx = du$$

$$= \int \frac{du}{u^2} = -\frac{1}{u} + C$$

$$= \boxed{\frac{-1}{(x-1)} + C}$$

j. $\int \frac{e^{2x}}{e^x+5}\,dx$

$= \int \frac{e^x \cdot e^x}{e^x+5}\,dx$

$e^x+5 = u \quad \rightsquigarrow e^x = u-5$

$e^x dx = du$

$= \int \frac{u-5}{u}\,du$

$= \int \cancel{\frac{u}{u}}\,1\,du - \int \frac{5}{u}\,du$

$= u - 5 \cdot \ln|u| + C$

$= (e^x+5) - 5 \cdot \ln|e^x+5| + C$

$= \boxed{e^x - 5 \cdot \ln|e^x+5| + C}$

k. $\int \frac{dx}{x(\sqrt{1-\ln^2 x})}$

$\ln x = u$

$1/x\,dx = du$

$\int \frac{du}{\sqrt{1-u^2}}$

$= \text{Bqsin}\ u + C$

$= \boxed{\text{Bqsin}\ \ln x + C}$

l. $\int e^{5x+1} \cdot \sin(1-x)\,dx$

$1-x = t \quad \rightsquigarrow -x = t-1$

$-dx = dt \qquad x = 1-t$

$dx = -dt$

$= -\int e^{5\cdot(1-t)+1} \cdot \sin t\,dt$

$= -\int e^{5-5t+1} \cdot \sin t\,dt$

$= -\int e^6 \cdot e^{-5t} \cdot \sin t\,dt$

$= -e^6 \boxed{\int e^{-5t} \cdot \sin t\,dt}^{A}$

$f(x) = e^{-5t} \longrightarrow f'(x) = -5 \cdot e^{-5t}$

$g(x) = -\cos t \longleftarrow g'(x) = \sin t$

$= -e^{-5t} \cdot \cos t - 5 \cdot \boxed{\int e^{-5t} \cdot \cos t \, dt}^{B}$

$f(x) = e^{-5t} \longrightarrow f'(x) = -5 \cdot e^{-5t}$

$g(x) = \sin t \longleftarrow g'(x) = \cos t$

$= \boxed{e^{-5t} \cdot \sin t - 5 \cdot \int \sin t \cdot e^{-5t} dt}^{B} \rightarrow$ idem als beginintegraal!

$\int e^{-5t} \cdot \sin t \, dt = -e^{-5t} \cdot \cos t - 5 \cdot \boxed{e^{-5t} \cdot \sin t + 5 \cdot \int \sin t \cdot e^{-5t} dt + C}^{B}$

$\int e^{-5t} \cdot \sin t \, dt = -e^{5t} \cdot \cos t - 5 e^{-5t} \cdot \sin t - 25 \int \sin t \cdot e^{-5t} dt + C$

$26 \int e^{-5t} \cdot \sin t \, dt = -e^{-5t} \cdot \cos t - 5 e^{-5t} \cdot \sin t + C$

$A \; \boxed{\int e^{-5t} \cdot \sin t \, dt} = -\frac{1}{26} \cdot e^{-5t} \cdot \left(\cos t + 5 \sin t \right) + C$

$= +\frac{1}{26} \cdot e^{6} \cdot e^{-5t} \cdot \left(\cos t + 5 \sin t \right) + C$

$= \frac{1}{26} \cdot e^{6} \cdot e^{-5 \cdot (1-x)} \left(\cos(1-x) + 5 \sin(1-x) \right) + C$

$= \frac{1}{26} \cdot e^{6} \cdot e^{-5} \cdot e^{5x} \cdot \left(\cos(1-x) + 5 \sin(1-x) \right) + C$

$= \boxed{\frac{1}{26} \cdot e^{1+5x} \cdot \left(\cos(1-x) + 5 \sin(1-x) \right) + C}$

m. $\int \frac{x - \text{Bgtan}\, x}{1+x^2} dx$

$= \int \frac{x}{1+x^2} dx - \int \frac{\text{Bgtan}\, x}{1+x^2} dx$

$1+x^2 = v \qquad \text{Bgtan}\, x = u$

$2x = dv \qquad 1/1+x^2 = du$

$x = 1/2 \, dv$

$= \frac{1}{2} \int \frac{dv}{v} - \int u \, du$

$$= \frac{1}{2} \ln|N| - \frac{1}{2} u^2 + C$$

$$= \boxed{\frac{1}{2} \ln|1+x^2| - \frac{1}{2} \cdot (\text{Arctan}\, x)^2 + C}$$

$$\int \frac{x}{\sqrt{(4+x^2)^3}} dx$$

$4+x^2 = u$

$2x\,dx = du$

$x\,dx = 1/2\, du$

$$= \frac{1}{2} \int \frac{du}{\sqrt{u^3}}$$

$$= \frac{1}{2} \cdot u^{-1/2} \cdot -2 + C$$

$$= \boxed{\frac{-1}{\sqrt{4+x^2}} + C}$$

OPLOSSINGEN

OEFENING 1:

$a)$ $]-\infty,1[\,\cup\,]2,+\infty[$

$b)$ $]2,+\infty[$

$c)$ $\mathbb{R}$

$d)$ $\mathbb{R}\setminus\{-4,0,4\}$

$e)$ $]3,+\infty[$

$f)$ $]2,4]$

$g)$ $]-2,1[\,\cup\,]1,+\infty[\,=\,]-2,+\infty[\,\setminus\{1\}$

$h)$ $[-\frac{3}{2},-\frac{1}{3}[\,\cup\,]\frac{1}{3},\ \frac{3}{2}]$

OEFENING 2:

$a)\left]-\infty,\dfrac{2}{3}\right[$

$b)$ $]5,+\infty[$

$c)$ $f(x)$ is injectief, dus ook inverteerbaar

$d)$ $f^{-1}(x)=\dfrac{-1}{3(x-5)^2}+\dfrac{2}{3}$

OEFENING 3:

$a)$ $\log_5 3=\frac{\log 3}{\log 5}$

$b)$ 1

$c)$ $1-\frac{1}{\sqrt[3]{3}}$, -8

OEFENING 4:

$a)$ $\{2\}$

$b)$ $\{9\}$

$c)$ $\{2\}$

$d)$ $\{3\}$

$e)$ $\{0{,}001; 100000\}$

$f)$ $\{\log_2 3\}$

OEFENING 5:

$a)$ $W = -\frac{1}{2}q^2 + 10q - 8$

$b)$ $q = 10$ en $W_{max} = 42$

$c)$ $q = 10 - \sqrt{84}$ en $q = 10 + \sqrt{84}$

$d)$ $q \in [0{,}25]$

OEFENING 6:

Aanbodcurve wordt $p = \frac{2}{5}q + 1$

Marktevenwicht bij $q = \cancel{10}\,4$, dus omzet is $p.q = \cancel{10.5 = 50}$ $\frac{52}{5}$

OEFENING ~~7~~ 10:

De functie is essentieel discontinu in 2 (wel linkscontinu in 2 want $LL = f(2) = 4$).

OEFENING 8:

$a)$ $LL = -\infty$; $RL = +\infty$	$b)$ 0
$c)$ $-\infty$	$d)$ 1
$e)$ $LL = 0$; $RL = +\infty$	$f)$ -1
$g)$ $RL = \sqrt{2}$; LL: /	$h)$ -2
$i)$ $LL = +\infty$; $RL = -\infty$	$j)$ $+\infty$
$k)$ $-\infty$	$l)$ $-\frac{5}{2}$

$m)\ \frac{7}{2}$

$n)\ LL = -\infty\, ;\, RL = +\infty$

$o)\ -\frac{1}{2}$

$p)\ 0$

<u>OEFENING 9:</u>

$a)$ VA: $x = 3$

SA: $y = 4x + 10$

$b)$ VA: $x = 0$

SA: $y = x - 5$

$c)$ SA: $y = x + \frac{1}{2}$ (voor $x \to +\infty$)

SA: $y = -x - \frac{1}{2}$ (voor $x \to -\infty$)

<u>OEFENING ~~10~~ 7:</u>

$$f(3) = \frac{9}{2}$$

<u>OEFENING 11:</u>

$a)\ y = \dfrac{1}{4}x + 1$

$b)\ y = \dfrac{\ln 2}{2}x + \dfrac{1}{2}(1 + \ln 2)$

$c)\ y = -5x + 18$

<u>OEFENING 12:</u>

$y = 3x + 4$

<u>OEFENING 13:</u>

Maximum in 2 met waarde 2. Minimum in 1 met waarde 0.

OEFENING 14:

$a)\ f'(x) = x^{x^2+1}(2\ln|x|+1)$

$$b)\ f'(x) = \frac{(\tan x)^{\sqrt[3]{x}}}{3\sqrt[3]{x^2}}\left[\ln|\tan x| + \frac{3x}{\sin x \cos x}\right]$$

$c)\ f'(x) = (\cos(3x+1))^{\sin x}\,[\cos x.\ln|\cos(3x+1)| - 3\tan(3x+1)\sin x]$

$$d)\ f'(x) = \left(\frac{x^2+3}{6x+2}\right)^x \left[\ln\left|\frac{x^2+3}{6x+2}\right| + x\left(\frac{2x}{x^2+3} - \frac{6}{6x+2}\right)\right]$$

OEFENING 15:

$$a)\ \varepsilon_y = 2x^2 + 1 + \frac{1}{\ln x} - \frac{x}{2(x+3)}$$

$b)\ \varepsilon_y = 5 - 2x + 3x\cot x$

$$c)\ \varepsilon_y = 3 + 5x + x\cot x - \frac{x(6x+1)}{3x^2+x-1}$$

OEFENING 16:

$a)\ LL = -\infty\,; RL = +\infty$

$b)\ 0$

$c)\ LL = 0\,; RL = +\infty$

$d)\ 0$

$e)\ LL = -\infty\,; RL = +\infty$

OEFENING 17:

b) HA: $y = 0$ en VA: $x = 0$ (enkel rechts)

a) HA: $y = 0$ (enkel op $-\infty$)

$c)$ SA: $y = -x - 1$ en VA: $x = -1$

OEFENING 18:

a) Afname van de vraag met $\frac{9}{32}\%$

b) We weten: $O'(p) = V(p).(1+\varepsilon_q(p))$ waarbij $V(p) \geq 0$

$\varepsilon_q(6) = -\frac{9}{8} < -1 \;\; \Rightarrow \;\; O'(p) < 0 \;\; \Rightarrow$ Opbrengsten zullen afnemen

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OEFENING 19:

Buigraaklijn in $x = -2$ met vergelijking $y = \frac{-1}{e^2}(x+4)$

OEFENING 20:

$a)$ $2\sqrt{x}\ln x - 4\sqrt{x} + C$

$b)$ $\dfrac{3\sqrt[3]{(2x+2)^5}}{20} - \dfrac{3\sqrt[3]{(2x+2)^2}}{4} + C$ → Niet vereenvoudigd! $\frac{3}{10}\cdot\sqrt[3]{(2x+2)^5} - \frac{3}{4}\cdot\sqrt[3]{(2x+2)^2} + C'$

$c)$ $\frac{1}{6}\sqrt{(x^4+3)^3} - \frac{3}{2}\sqrt{x^4+3} + C$

$d)$ $2(\sqrt{x}+1)\,\text{Bgtan}(\sqrt{x}+1) - \ln\left(1+(\sqrt{x}+1)^2\right) + C$

$e)$ $-\ln|\cos(\ln x)| + C$

$f)$ $(2x - 4x^2 + 32)\sin\frac{x}{2} + (4-16x)\cos\frac{x}{2} + C$

$g)$ $-\cos x.e^{\cos x} + e^{\cos x} + C$ $= e^{\cos x}\cdot(-\cos x + 1 + C')$

$h)$ $\frac{1}{4}(x^4+4x)\,\text{Bgsin}(x^4+4x) + \frac{1}{4}\sqrt{1-(x^4+4x)^2} + C$

$i)$ $-\dfrac{1}{x-1} + C$

$j)$ $e^x - 5\ln(e^x+5) + C$

$k)$ $\text{Bgsin}(\ln x) + C$

$l)$ $\frac{1}{26}e^{5x+1}\cos(1-x) + \frac{5}{26}e^{5x+1}\sin(1-x) + C$ $= \frac{1}{26}\, e^{5x+1}\cdot(\cos(1-x) + 5\cdot(\sin(1-x)) + C$

$m)$ $\frac{1}{2}\ln(1+x^2) - \frac{1}{2}\,\text{Bgtan}^2 x + C$

$n)$ $-\dfrac{1}{\sqrt{4+x^2}} + C$